

EXAMINATION	NATIONAL SENIOR CERTIFICATE
GRADE	12
DATE	JUNE 2024
SUBJECT	MATHEMATICS
PAPER	1
MARK TOTAL	150
DURATION (HOURS)	3
NUMBER OF PAGES	12



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INSTRUCTIONS

Read the following instructions carefully before answering the questions.

1. This paper consists of 9 questions. Answer all the questions.
2. Clearly show **ALL** calculations, diagrams, graphs etc. that you have used in determining your answers.
3. Answers only will not necessarily be rewarded full marks.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. An information sheet with formulae is included at the end of the question paper.
8. Write neatly and legibly.

QUESTION 1

1.1

1.1.1 Solve for x : $x^2 - 3x - 10 = 0$ (2)

1.1.2 HENCE solve for $p - 3\sqrt{p} - 10 = 0$ if $p \in \mathbb{R}$. (3)

1.2 Solve the following:

1.2.1 $2x^2 - 3x - 8 = 0$
(Give the answer correct to two decimal places) (3)

1.2.2 $80 - x^2 < 2x$ (3)

1.3 Solve for x and y simultaneously:

$$\begin{aligned} 2x + y &= 5 \\ 4x^2 - 2y^2 &= 46 \end{aligned}$$
 (6)

1.4 Calculate the value of $\sqrt{\frac{5^{1005} + 5^{1007}}{26(5^{1003})}}$ (4)

1.5 In a quadratic equation, $ax^2 + bx + c = 0$, the values of a, b and c are all real and negative.It is given that a, b and c , in this order, forms a geometric sequence. (4)

Show that the given equation has no real roots.

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QUESTION 2

2.1 If $x + 2$; $4x$; $6x + 4$ forms an arithmetic sequence, determine the sequence. (3)

2.2 Given

$$\sum_{k=1}^n \frac{5}{3} (3)^{k-1} = \frac{1820}{3}$$

Determine the value of n . (6)

2.3 The geometric series below is convergent:

$$1 + \frac{2x - 5}{2} + \left(\frac{2x - 5}{2}\right)^2 + \dots$$

2.3.1 Determine the possible value(s) of x . (2)

2.3.2 If the sum to infinity of the series is $\frac{4}{9}$, determine the value of x . (5)

[16]

QUESTION 3

- 3.1 The general term in a sequence with a constant second difference is given by
 $T_n = dn^2 + kn + p$.

Prove that the second difference is $2d$. (4)

- 3.2 Given the sequence

3; 9; 27; ...

Louisa says that the fourth term is **81**. Thabo differs from her and says that the fourth term is **57**.

3.2.1 Explain why both Louisa and Thabo can be correct. (2)

3.2.2 Determine the 10th term of Thabo's sequence. (4)

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**QUESTION 4**

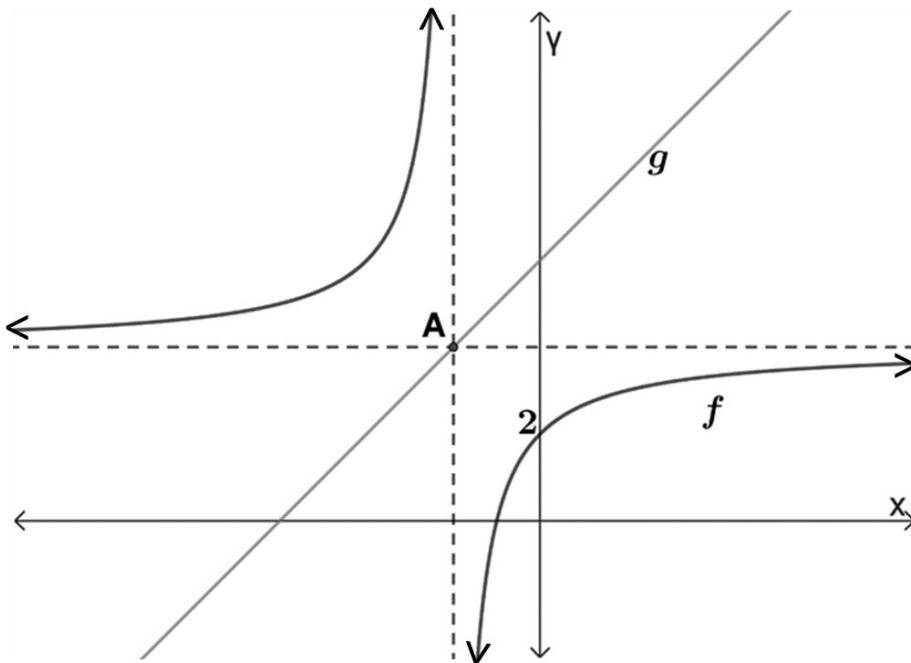
- 4.1 A company has machinery that costs R940 000. After 7 years they decided to replace the machinery. The machinery depreciates annually at a rate of $d\%$ p.a. on the reduction balance method. Inflation on the machinery is 12% p.a. They decide to trade in the old machinery for the new ones.
- 4.1.1 Calculate the value of d if they will need R1 843 712,18 to buy the new machinery. (5)
- 4.1.2 They decide to set up a sinking fund to buy the machinery. They negotiate with the bank and decide to pay a fixed monthly amount into the account. The bank will give them an interest rate of 9% p.a. compounded monthly. They will start saving in one month's time and the last payment will be after 7 years. Calculate their monthly payment. (2)
- 4.2 Loyiso buys computer equipment for R22 000. He is required to repay the debt by means of equal monthly payments. The interest rate is 15% p.a. compounded monthly. His first payment will be in two month's time, but the loan must be repaid 4 years after he buys it.
- 4.2.1 Calculate how much he would pay in total for the computer equipment at the end of 4 years. (4)
- 4.2.2 Exactly 2 years after the transaction, Loyiso wins R12000 in a competition. Will he be able to settle his debt with the money he won? Show all calculations. (5)

[16]

QUESTION 5

5.1 The diagram below shows the graphs of $f(x) = \frac{a}{x+p} + q$ and $g(x) = x + c$.

- ❖ The graph of f cuts the y -axis at $(0,2)$.
- ❖ The asymptotes of f intersect at $A(-2;4)$.
- ❖ The graph of $g(x)$ represents the axis of symmetry of f .



5.1.1 Determine the values of a, p and q . (4)

5.1.2 Determine the equation of the axis of symmetry. (2)

5.2 Given $f(x) = \left(\frac{3}{2}\right)^x$

5.2.1 Give the equation of $g(x)$ if g is the reflection of f in the y -axis. (1)

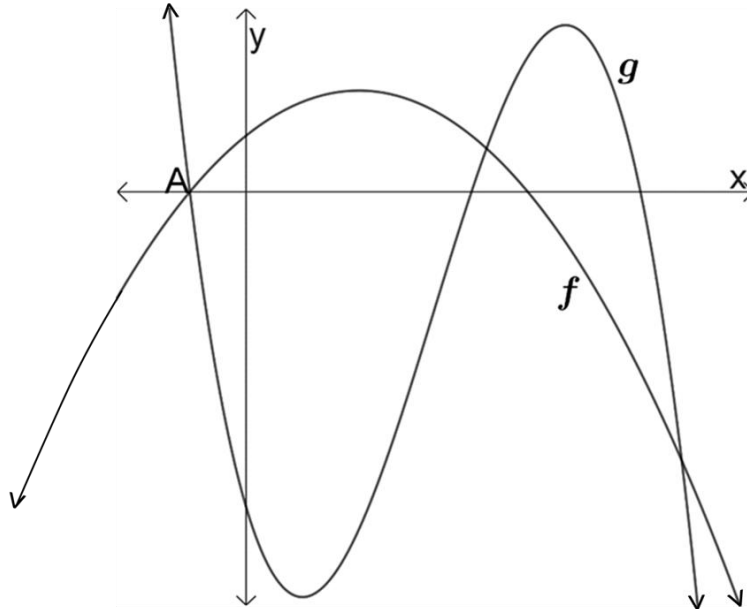
5.2.2 Give the equation of the inverse of f in the form $f^{-1}(x) = \dots$ (2)

5.2.3 Draw graphs of f, g and f^{-1} on the same set of axes. Clearly show the names of the graphs and the intercepts with the axes. (3)

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QUESTION 6

The diagram below shows the graphs of $f(x) = -x^2 + 4x + 5$ and $g(x) = -x^3 + 10x^2 - 17x - 28$. The graphs intersect at A, that lies on the x -axis, and at two other points.



- 6.1 Rewrite the equation of f in the form $f(x) = a(x + p)^2 + q$ (3)
- 6.2 Write down the turning point of f . (2)
- 6.3 Calculate the x - intercepts of both graphs. (7)
- 6.4 Calculate the x -values of the turning points of g . (3)
- 6.5 Give the equation of $h(x)$ if h is the graph formed if f translates 2 units to the right and 4 units up. (2)
- 6.6 Use the graph and determine the values of x for which $f(x) \cdot g(x) > 0$. (3)
- 6.7 Write down for which single value of x is it true that $g''(x) < g'(x) < g(x)$.
Motivate your answer. (3)

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QUESTION 7

7.1 If $f(x) = 3x^2 + 1$, determine $f'(x)$ from first principles. (5)

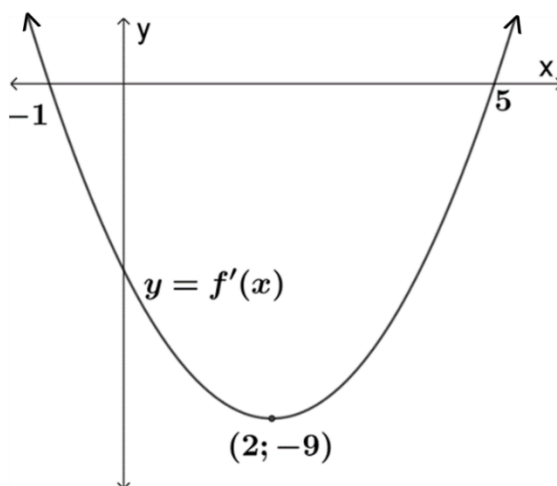
7.2 Differentiate the following:

7.2.1 Determine $f'(x)$ if $f(x) = 2x - \sqrt{x}$ (3)

7.2.2 Determine $D_x \left[\frac{x^2 - 3x + 2}{x - 1} \right]$ (3)

7.3 Given $f(x) = x^3 - 5x^2 + 2x - 1$. Determine the equation of the tangent to f at the point $(2; -9)$. (5)

7.4 The graph below represents the function of $f'(x)$, the DERIVATIVE of $f(x) = ax^3 + bx^2 + cx + 4$.



7.4.1 Write down the x -value where f will have a maximum. Give reasons for your answer. (3)

7.4.2 Write down x -value of the point of inflection of f . Give a reason for your answer. (2)

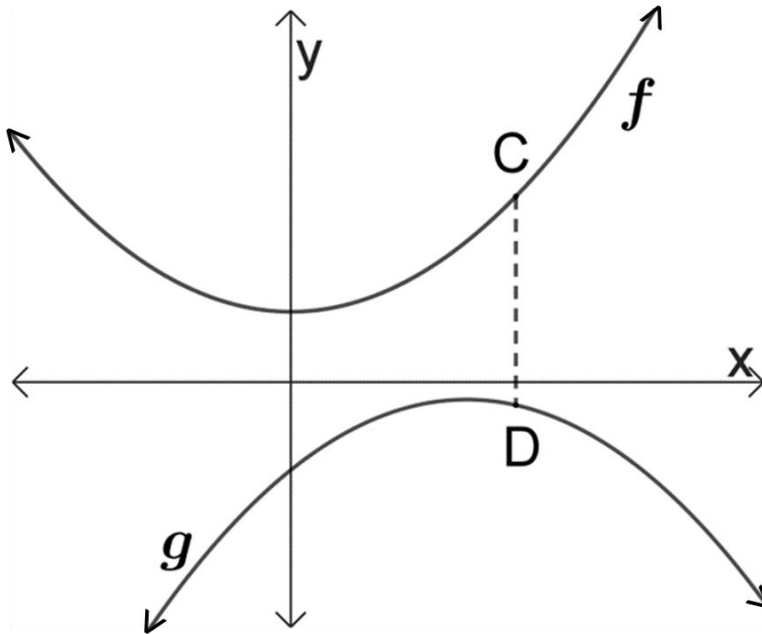
7.4.3 Determine the equation of $y = f'(x)$ (3)

7.4.4 Write down the derivative of $f(x) = ax^3 + bx^2 + cx + 4$, and hence determine the values of a, b and c (4)

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QUESTION 8

The sketch below shows the graphs of $f(x) = x^2 + 4$ and $g(x) = -x^2 + ax - 5$. The point C lies on f and the point D lies on g . The line CD is perpendicular to the x -axis.



The minimum length of CD is 7 units. Determine the value of a .

(7)

[7]

QUESTION 9

9.1 A survey of 80 people showed the following preference regarding vehicles:

- ❖ 44 preferred Toyota
- ❖ 35 preferred Ford
- ❖ 39 preferred Volkswagen
- ❖ 23 preferred Toyota and Volkswagen
- ❖ 19 preferred Ford and Volkswagen
- ❖ 20 preferred Ford and Toyota
- ❖ 9 preferred all three

9.1.1 Represent the information above in a Venn-diagram. (5)

Use the Venn-diagram to answer the following questions:

9.1.2 How many people preferred none of these vehicles? (1)

9.1.3 What is the probability that a random chosen person will prefer exactly one of these vehicles? (2)

9.2 A group of 10 children are wearing brightly coloured tracksuits and are standing randomly in a row.

9.2.1 In how many different ways can they be standing if it does not matter in which order they are standing? (1)

9.2.2 Two of the children are wearing red tracksuits, three are wearing blue tracksuits and the rest are wearing green tracksuits. What is the probability that the children wearing the same colour tracksuits will stand next to each other? (4)

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GRAND TOTAL [150]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$