

<b>EXAMINATION</b>		<b>NATIONAL SENIOR CERTIFICATE</b>	
<b>GRADE</b>		12	
<b>DATE</b>		NOVEMBER 2024	
<b>SUBJECT</b>		MATHEMATICS	
<b>PAPER</b>		1	
<b>MARK TOTAL</b>		150	
<b>DURATION (HOURS)</b>		3	
<b>NUMBER OF PAGES</b>		10	



SOUTH AFRICAN COMPREHENSIVE ASSESSMENT INSTITUTE  
SUID-AFRIKAANSE KOMPREENSIEWE ASSESSERINGSINSTITUUT

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This paper consists of nine questions. Answer all the questions.
2. Clearly show **ALL** calculations, diagrams, graphs etc. that you have used in determining your answers.
3. Answers only will not necessarily be awarded full marks.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. An information sheet with formulae is included at the end of the question paper.
8. Write neatly and legibly in BLOU ink.

**QUESTION 1**1.1 Solve for  $x$ :

1.1.1  $x^2 - 8x = 0$  (2)

1.1.2  $3x^2 + 2 = 9x$   
(Give the answers correct to two decimal places.) (4)

1.1.3  $2x^2 - 3x - 5 \leq 0$  (3)

1.2 Solve the following simultaneous equations:

$$x - 2y - 3 = 0 \quad \text{and} \quad 4x^2 + 6y = 5xy + 3$$
 (7)

1.3 Determine the value(s) of  $x$  if

$$\sqrt{3x + 1} - \frac{4}{\sqrt{3x + 1}} - 3 = 0$$
 (5)

1.4 Solve for  $x$ :  $5^{x+1} - 5^{x-1} = 120\sqrt{5}$  (3)

1.5 For which value(s) of  $k$  will the equation  $3x^2 - 5x + k = 0$  always have real roots? (4)

**[28]**

**QUESTION 2**

2.1 Consider the following arithmetic sequence 8; 15; 22;....

2.1.1 Determine the 34<sup>th</sup> term. (3)

2.1.2 Determine the sum of the first 40 terms. (3)

2.2 Calculate the value of

$$\sum_{k=1}^8 \frac{(2)^{k+1}}{3^k}$$

correct to two decimal digits. (5)

2.3 A ball is rolled repeatedly, on a flat surface, in a straight line attempting to reach a hole that is 13m away. The first roll covers a distance of 5m. The second roll starts where the first roll ended and reaches a distance that is 60% of the distance reached after the first roll. The third roll starts where the ball landed after the second roll and reaches a distance of 60% of the preceding roll. If this process is continued indefinitely in the same pattern, calculate whether the ball will reach the hole. (4)

**[15]**

**QUESTION 3**

Given the following quadratic sequence: 1;  $k$ ;  $3k + 1$ ;  $8k$ ; ...

3.1 Determine the value of  $k$ . (4)

3.2 Determine the general term of the sequence. (4)

**[8]**

## QUESTION 4

- 4.1 Lizzy bought a car for R200 000. The vehicle depreciates at 15% p.a. on the reducing balance method. The current value of the car is R105 400. Calculate for how many years she kept the car. (4)
- 4.2 Mashati buys a house. He gets a loan of R2 000 000 at an interest rate of 11% p.a. compounded monthly. He must pay back the loan after 20 years in equal monthly payments. The first payment is due one month after the loan is paid.
- 4.2.1 Calculate Mashati's monthly payment. (3)
- 4.2.2 Mashati cannot pay the 90<sup>th</sup> and 91<sup>st</sup> payments. Calculate his outstanding balance after 91 months, two months after his 89<sup>th</sup> payment. (5)
- 4.2.3 Mashati pays R21000 per month from the 92<sup>nd</sup> month to the bank. Calculate how many payments are needed to settle the loan. (3)

[15]

## QUESTION 5

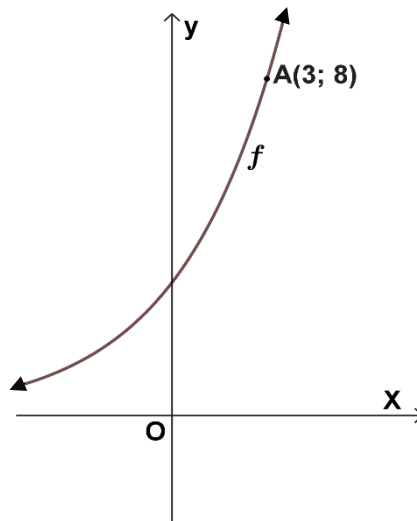
Given  $f(x) = x^2 - 4x - 5$  and  $g(x) = \frac{x+3}{x-1}$ .

- 5.1 Write  $g(x)$  in the form  $g(x) = \frac{a}{x-p} + q$ . (3)
- 5.2 If  $g(x) = \frac{4}{x-1} + 1$ , determine the intercepts with the axes of  $f$  and  $g$  (4)
- 5.3 Sketch the graphs of  $f$  and  $g$  on the same system of axes. Clearly show all intercepts with the axes, turning points and asymptotes. (4)
- 5.4 B is a point on  $f$  where  $f$  and the vertical asymptote of  $g$  intersect. Determine the coordinates of B. (2)
- 5.5 A tangent is drawn to  $f$ . The gradient of this tangent is  $-2$ . Show that the point on the graph of  $f$  where the tangent touches the graph, is the point B. (4)

[17]

## QUESTION 6

6.1 The sketch beneath shows the graphs of  $f(x) = a^x$ .

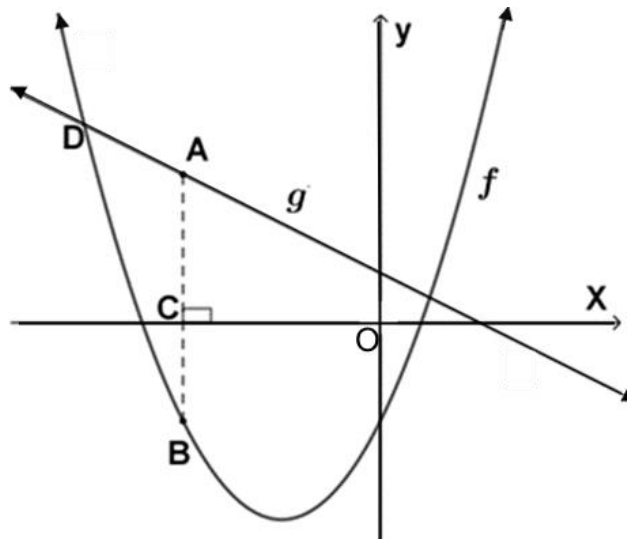


6.1.1 A(3; 8) is a point on  $f$ . Determine the value of  $a$ . (3)

6.1.2 Give the equation of the new graph,  $h(x)$ , if  $h$  is the graph of  $f$  which is translated 1 unit to the right and 3 units down. (2)

6.1.3 Give the equation of the reflection of  $f$  around the line  $y = x$ . (3)

6.2 The sketch beneath shows the graphs of  $f(x) = 2x^2 + 4x - 2$  and  $g(x) = -x + 1$ .



6.2.1 D is an intercept of  $f$  and  $g$ . Determine only the  $x$ -value of D. (4)

6.2.2 The point A lies on  $g$  and point B lies on  $f$ . AB intersects the  $x$ -axis at C. ACB is perpendicular to the  $x$ -axis. If  $AB = 5$  units, calculate the length of OC. (6)

**[18]**

**QUESTION 7**

7.1 If  $f(x) = -5x^2$ , determine the derivative  $f'(x)$  from first principals. (5)

7.2 Determine the following derivative:

$$\frac{d}{dx} \left( 8 - \frac{4}{x^2} + \sqrt[5]{x^4} \right) \quad (4)$$

7.3 The following equation is given:

$$f(x) = 4x^3 - 20x^2 + 25x$$

7.3.1 Calculate the intercepts of  $f$  with the axes. (2)

7.3.2 Determine the coordinates of the turning points of  $f$ . (4)

7.3.3 Sketch the graph of  $f$ . Clearly show the intercepts with the axes and the turning points. (3)

7.3.4 Determine the  $x$ -value of the point of inflection. (2)

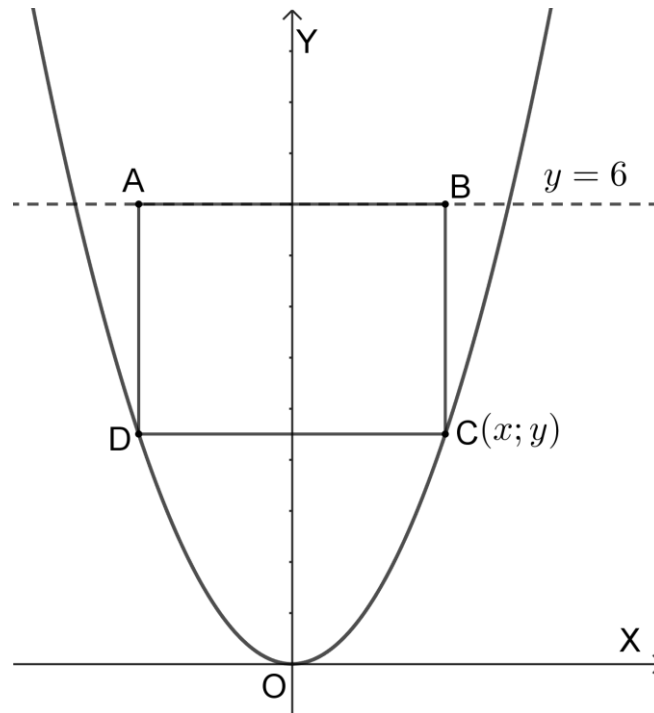
7.3.5 For which values of  $x$  is the graph concave up? (2)

7.3.6 The graph  $g(x) = f(x) + k$  has 3 separate  $x$ -intercepts. Give the values of  $k$ . (3)

**[25]**

### QUESTION 8

The sketch below shows the graph of  $y = x^2$  with a rectangle ABCD drawn inside.  
 The side AB of the rectangle lies on the line  $y = 6$ .  
 C( $x; y$ ) and D are points on the graph of  $y = x^2$ .



- 8.1 Write down the length of DC and BC in terms of  $x$ . (3)
- 8.2 Show that the area of the rectangle can be given as  $A(x) = 12x - 2x^3$ . (4)
- 8.3 Hence determine the maximum area of the rectangle. (4)

**[11]**

**QUESTION 9**

9.1 In a survey among 1500 mountain climbers, they were asked if they had ever fallen more than 2m. The results of the survey:

	Had fallen further than 2 m	Had never fallen further than 2 m	Total
Male	424	$b$	780
Female	$a$	$c$	720
	872	628	1500

9.1.1 Calculate the values of  $a$ ,  $b$  and  $c$ . (3)

9.1.2 Calculate the probability that a randomly selected person, who had fallen more than 2 m, is a female. (2)

9.1.3 Let event A = "A female" and event B = "A climber that had fallen more than 2m". Are A and B independent events? Motivate your answer with calculations. (4)

9.2 Eight people, consisting of four pairs, must be seated around a table. Calculate the probability that all the pairs will sit next to each other. Give the answer as a fraction. (4)

[13]

**GRAND TOTAL: [150]**

**END OF PAPER**



## INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1 + i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1 + i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$\text{In } \triangle ABC: \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum fx}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$