MARKING GUIDELINES

EXAMINATION	NATIONAL SENIOR CERTIFICATE
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SOUTH AFRICAN COMPREHENSIVE ASSESSMENT INSTITUTE

SUID-AFRIKAANSE KOMPREHENSIEWE ASSESSERINGSINSTITUUT

QUESTION 1

To promote blood donation a blood donor centre was established at a shopping centre. The table provided below displays the daily count of blood units donated in TEN days by the people at the shopping centre.

Days	1	2	3	4	5	6	7	8	9	10
Units of blood	45	59	65	73	79	85	91	99	101	110

1.1.1 Calculate the mean of the units of blood donated per day over the period of 10 days. (2)

$$\bar{x} = \frac{807}{10} = 80{,}7$$

✓✓ Answer

1.1.2 Determine the standard deviation of the data. (2)

$$\sigma = 19{,}45$$

✓✓ Answer

1.1.3 How many days was the number of units of blood donated at the shopping centre outside one standard deviation from the mean? (3)

Upper limit $= 80{,}7 + 19{,}45 = 100{,}15$

Lower limit $= 80{,}7 - 19{,}45 = 61{,}25$

$(61{,}25\,;100{,}15)$

Number of days $= 5$

✓ mean+ 1SD
✓ mean-1SD
✓ Answer

Correct answer only: full marks, provided that 1.1.1 and 1.1.2 both correct

1.2.1 Draw a box and whisker diagram to represent the number of units of blood donated by people at the shopping centre. (3)

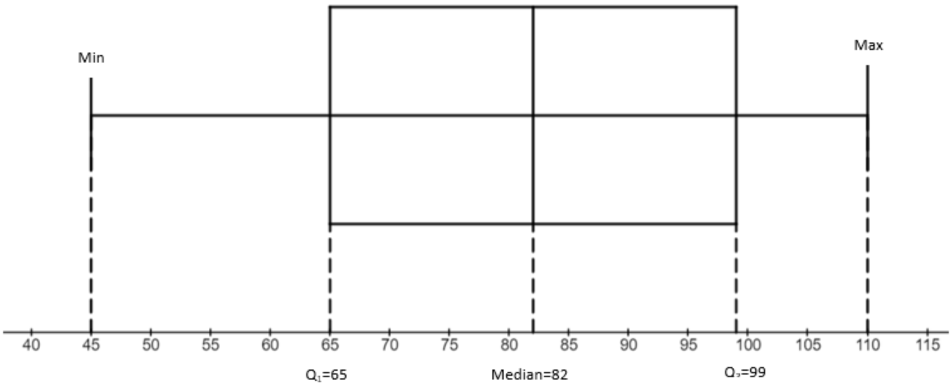
Min = 45 ✓ Min & Max values

Q_1 = 65 ✓ Q_1 & Q_3

Median = 82 ✓ Median & scale

Q_3 = 99

Max = 110



1.2.2 Describe the skewness of the data. Motivate your answer. (2)

Data is slightly negatively skewed. ✓ Answer

The mean is less than the median; 80,7 < 82 ✓ Motivation

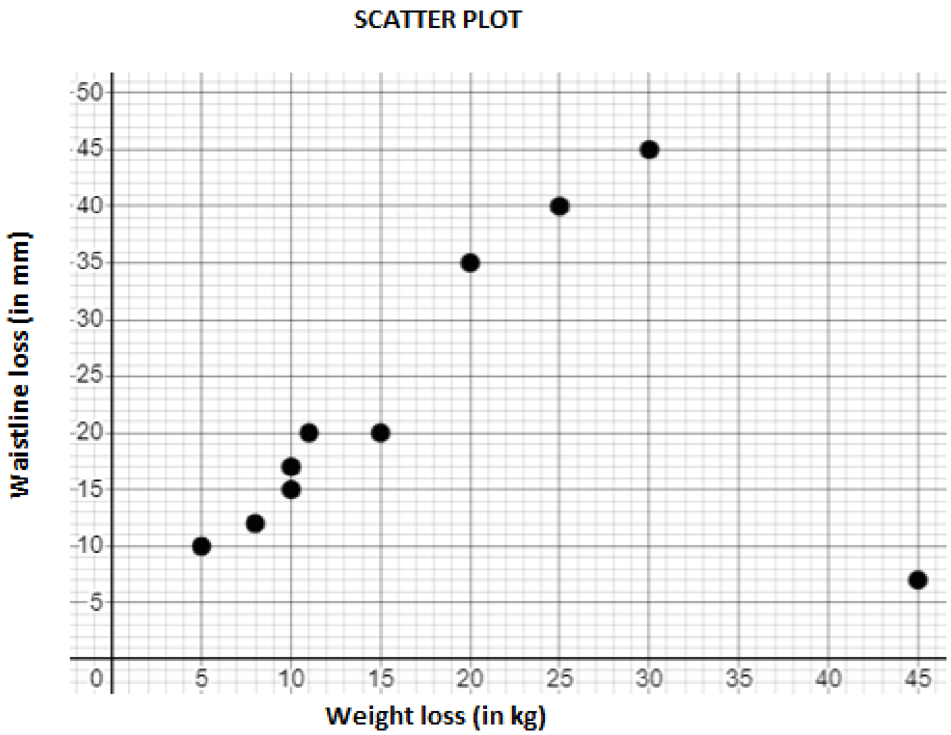
[12]

QUESTION 2

The table below represents the weight loss and waistline loss of 10 contestants who took part in a weight loss competition.

Weight loss (in kg) (x)	**5**	**10**	**45**	**8**	**10**	**15**	**25**	**30**	**11**	**20**
Waistline loss (in mm) (y)	**10**	**15**	**7**	**12**	**17**	**20**	**40**	**45**	**20**	**35**

A scatter plot of the above results is shown below.



2.1 Identify an outlier in the data above. (1)

$(45; 7)$ ✓ Answer

2.2 Determine the equation of the least squares regression line for the data. (2)

$A = 16{,}98$ ✓ $A = 16{,}98$

$B = 0{,}29$ ✓ $B = 0{,}29$

$y = 0{,}29x + 16{,}98$

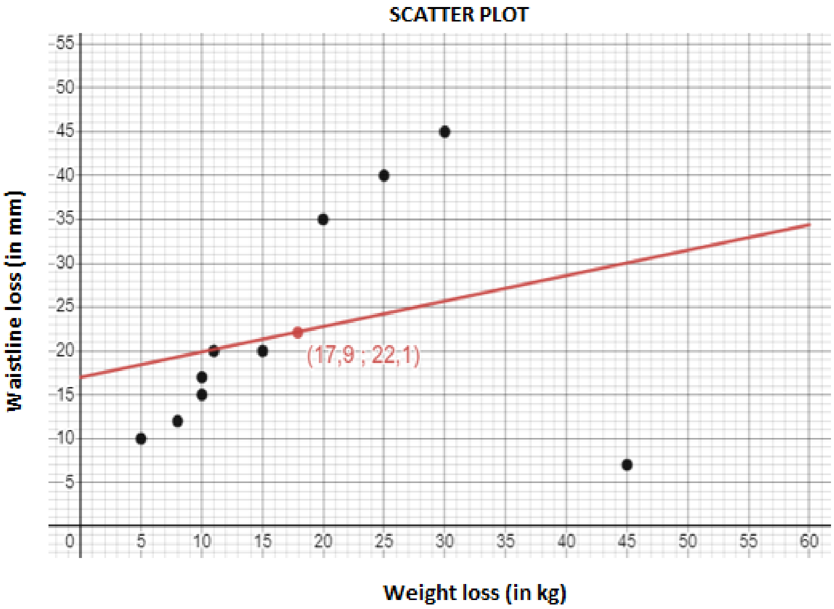
2.3 If a contestant lost 21,2 kg during the same time. Predict the waistline loss of this contestant, correct to two decimal places. (2)

$y = 0{,}29x + 16{,}98$ ✓ Subst $x = 21{,}2$

$y = 0{,}29(21{,}2) + 16{,}98$

$y = 23{,}13$ mm ✓ Answer

2.4 Draw the least square regression line on the scatter plot **provided above**. (3)



✓ y-intercept (0;16,98)

✓(17,9; 22,1)

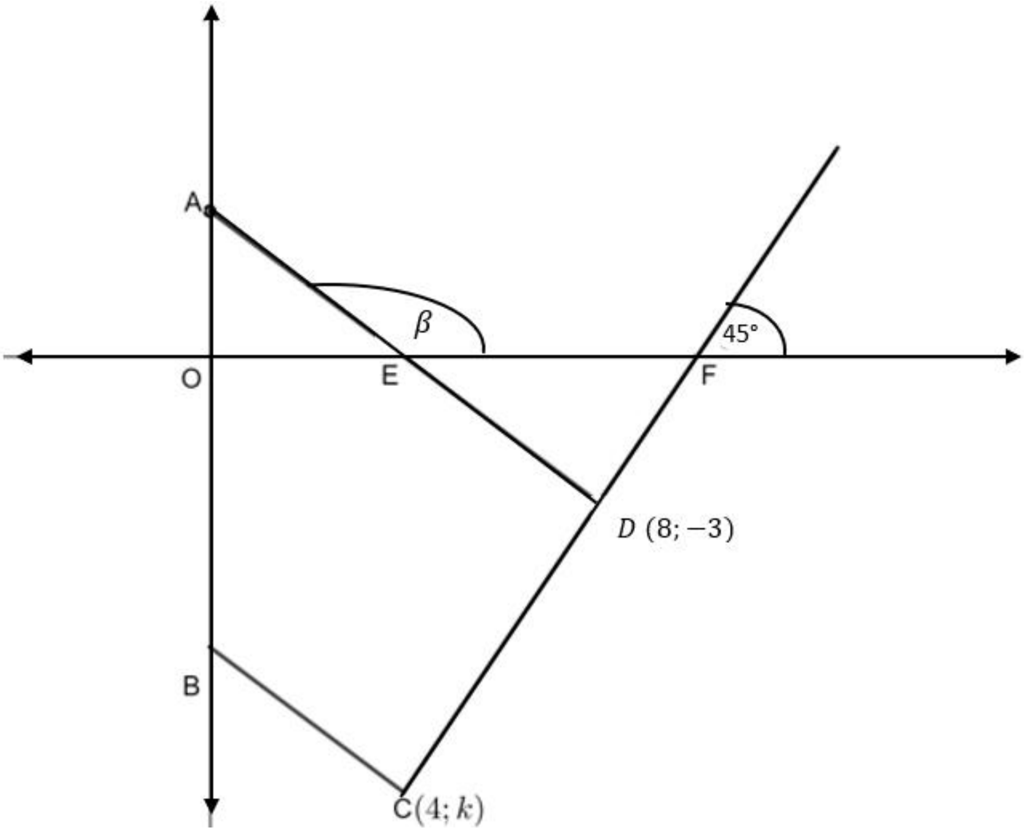
✓ no x-intercept;

[8]

QUESTION 3

In the diagram below:

- AD is a straight line with the equation $y = -\frac{3}{4}x + 3$.
- E and A are the x – and y –intercepts respectively of AD .
- The angle of inclination of AD is β.
- From $C(4; k)$ a line, parallel to AD is drawn that intersects the y –axes at B.
- Line CD is drawn through $D(8; -3)$ and intersects the x-axis at F.
- The angle of inclination of CD is $45°$.



3.1 Determine the coordinates of A and E. (4)

$A(0; 3)$ ✓ A(0;3)

E-x –intercept; $y = 0$ ✓ y=0

$0 = -\frac{3}{4}x + 3$ ✓ Simplify

$x = 4$ ✓ E (4;0)

$E(4; 0)$

3.2 Show that $k = -7$. (3)

$m_{CD} = \tan 45° = 1$ ✓ mCD=1

$m_{DF} = \frac{k-(-3)}{4-8} = 1$ ✓ Subst (8;-3)

$k + 3 = -4$ ✓ Subst (4;k)

$k = -7$

OR

OR:

$m_{CD} = \tan 45° = 1$ ✓ mCD=1

$\therefore y = mx + c$ OR $y - y_1 = m(x - x_1)$ ✓ Sub (8;-3)

$-3 = (1)(8) + c$ $y + 3 = (1)(x - 8)$

$c = -11$ $y = x - 11$ ✓ Sub (4;k)

$y = x - 11$

$\therefore k = x - 11$

$k = 4 - 11$

$\therefore k = -7$

3.3 Determine the equation of BC. (3)

Given AD ∥ BC, $m_{AD} = m_{BC}$ ✓ $m_{BC} = -\frac{3}{4}$

$\therefore m_{BC} = -\frac{3}{4}$

✓ Subst (4;-7)

$y = mx + c$ OR $y - y_1 = m(x - x_1)$

$-7 = \left(-\frac{3}{4}\right)(4) + c$ $y + 7 = \left(-\frac{3}{4}\right)(x - 4)$ ✓ Answer

$c = -4$

$\therefore y = -\frac{3}{4}x - 4$ $y = -\frac{3}{4}x - 4$

3.4 Determine whether ΔDEF is right-angled. Show all your calculations. (4)

$m_{DF} = \tan 45° = 1$ ✓ $m_{DF}=1$

$m_{AD} = -\frac{3}{4}$

✓ $m_{AD} \times m_{D\dot{F}}$

$m_{AD} \times m_{D\dot{F}} = -\frac{3}{4} \times 1$ ✓ $\neq -1$

✓ Conclusion

$m_{AD} \times m_{D\dot{F}} = -\frac{3}{4} \neq -1$

$\therefore$ ΔDEF is not right-angled.

3.5 Calculate the area of △DEF. (5)

CF:

$y = mx + c$ OR $y - y_1 = m(x - x_1)$

$-3 = (8) + c$ $\quad y - (-3) = x - 8$

$c = -11$ $\quad y = x - 11$

$y = x - 11$

$F(11;0)$

$EF = 11 - 4 = 7$

$DF = \sqrt{(11-8)^2 + (0-(-3))^2}$

$DF = 3\sqrt{2}$

$Area = \frac{1}{2} EF \cdot DF \sin 45°$

$Area = \frac{1}{2}(7)(3\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)$

$Area = \frac{21}{2} \quad or\ 10{,}5\ unit^2$

✓ F(11;0)
✓ $EF = 7$
✓ $DF = 3\sqrt{2}$
✓ Corr. Subst in area rule
✓ Answer

3.6 Let G be a point in the fourth quadrant such that CEDG forms a parallelogram. Calculate the coordinates of G. (4)

$E(4;0)$

Diagonals of parm bisect each other.

Midpoint CD: $\left(\frac{8+4}{2} ; \frac{-3-7}{2}\right) = (6 ; -5)$

∴ midpoint EG $= (6;-5) = \left(\frac{x+4}{2} ; \frac{y+0}{2}\right)$ . . .

$\therefore \frac{x+4}{2} = 6$ and $\frac{y+0}{2} = -5$

$x = 8$ $\quad y = -10$

$\therefore G\ (8 ; -10)$

CA from 3.1
✓ midpoint CD value
✓ midpoint EG value

OR

✓ $4 + 4$
✓ $-3 - 7$
✓ $x = 8$
✓ $y = -10$

OR:

Translation:
From E to D x-values: 4
From D to g y-values: 7 down
$G : (4 + 4 ; -3 - 7)$

$\therefore G\ (8; -10)$

OR:

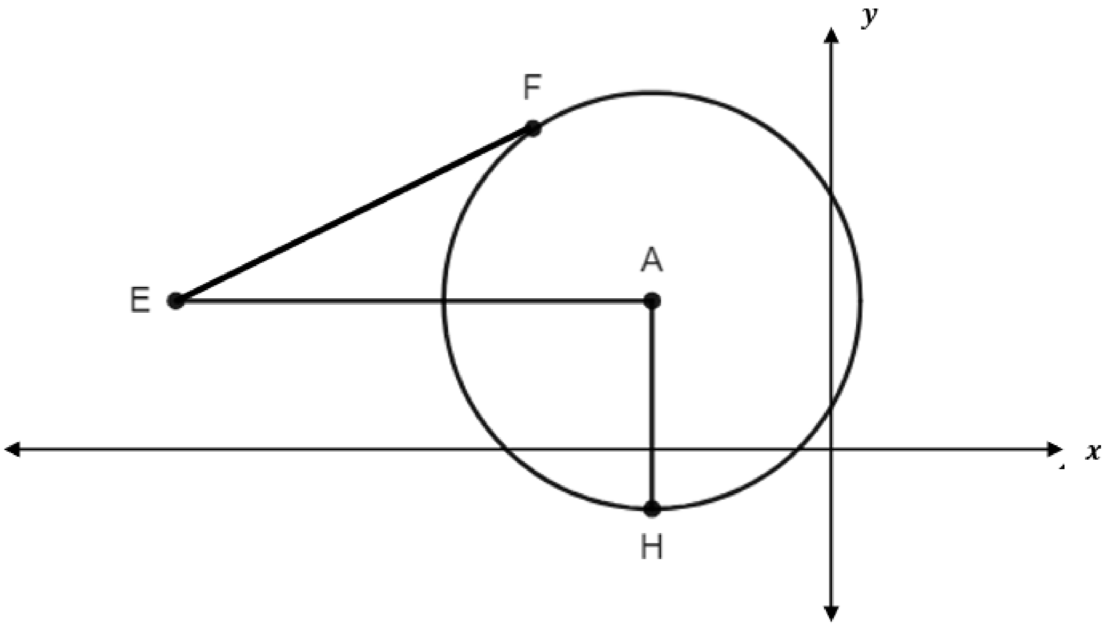
C and E has same x-values, $\therefore EC \perp x - axis$
D and E should have same x-values as $EC \parallel DG$,
$x_G = 8$
From D to g y-values: 7 down

$\therefore G(8; -10)$

[23]

QUESTION 4

In the diagram circle with centre A and equation $x^2 + y^2 + 12x - 10y = -12$ is given.
AE is parallel to the x-axes and AH is perpendicular to the x –axes.
EF is a tangent to the circle such that $EF = \sqrt{95}$.



4.1 Determine the coordinates of A and the radius of the circle. (5)

$x^2 + 12x + (6)^2 + y^2 - 10y + (5)^2 = -12 + 36 + 25$

✓ $(x + 6)$

$(x + 6)^2 + (y - 5)^2 = 49$ ✓ $(y - 5)$

✓ 49

A = $(-6\,;\ 5)$ ✓ A(-6;5)

Radius = 7 ✓ r=7

4.2 Determine the equation of the tangent to the circle at point H. (1)

$y = -2$ ✓ Answer

4.3 Determine the coordinates of E. (3)

$EA = \sqrt{(\sqrt{95})^2 + (7)^2}$ . . . Pythagoras

$EA = 12$

$E\ (-6 - 12\ ;5)$

$= E(-18; 5)$

✓ Using Pythagoras
✓ EA=12
✓ E(-18;5)

OR

OR:

$EA = \sqrt{(\sqrt{95})^2 + (7)^2}$ . . . Pythagoras

$EA = 12$

$(12)^2 = (x + 6)^2 + (5 - 5)^2$

$144 = x^2 + 12x + 36$

$x^2 + 12x - 108 = 0$

$(x - 6)(x + 18) = 0$

$x = 6$ or $x = -18$

$\therefore E(-18\ ;5)$

✓ Using Pythagoras
✓ distance formulae
✓ E(-18;5)

4.4 Calculate the size of $F\hat{E}A$. (2)

$\tan F\hat{E}A = \frac{7}{\sqrt{95}}$

$F\hat{E}A = 35{,}69°$

✓ tan ratio

✓ Answer

OR

OR

$\sin F\hat{E}A = \frac{7}{12}$

$F\hat{E}A = 35{,}69°$

✓ sin ratio

✓ Answer

OR

OR

$\cos F\hat{E}A = \frac{\sqrt{95}}{12}$

$F\hat{E}A = 35{,}69°$

✓ cos ratio

✓ Answer

4.5 A circle with centre B and represented by the equation $(x+10)^2+(y-5)^2=k$, touches circle A internally.
Calculate the value of k. (2)

A (-6 ; 5)
B (-10; 5)

✓ AB=4

$AB=\sqrt{(-10+6)^2+(5-5)^2}$
$AB=4$ units

$r_A - r_B = AB$
$7-r=4$
$r=3$

✓ k=9

$$\therefore k=r^2=9$$

4.6 Circle A is reflected about the x-axis and translated 2 units to the right to form a new circle with centre P.
Write down the equation of circle P. (2)

$(x+6-2)^2+(y+5)^2=49$ ✓ $(x+4)^2$
$(x+4)^2+(y+5)^2=49$ ✓ $(y+5)^2$

[15]

QUESTION 5

5.1 If $17\cos\theta = -8$ and $0° < \theta < 180°$, calculate, **without the use of a calculator**:

5.1.1 $\tan(360° - \theta)$ (3)

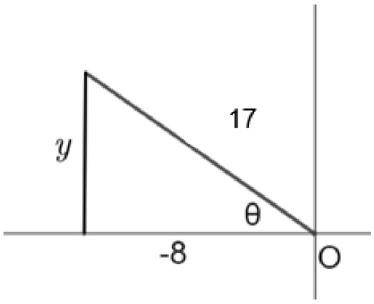
$\cos\theta = -\frac{8}{17}$

$y = \sqrt{(17)^2 - (8)^2}$

$y = 15$

$\tan(360° - \theta) = -\tan\theta$

$= -\left(\frac{15}{8}\right)$



✓ $\cos\theta = -\frac{8}{15}$

✓ $y = 15$

✓ Answer

5.1.2 $\sin 2\theta$. (3)

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{15}{17}\right)\left(-\frac{8}{17}\right)$

$= -\frac{240}{289}$

CA from 6.1.1
✓ Identity
✓ $\sin\theta$
✓ Answer
(not decimal)

5.1.3 $\sin(810° - \theta)$. (3)

$\sin(810° - \theta) = \sin(90° - \theta)$ ✓ sin (90°-θ)

$= \cos\theta$ ✓ cos α

$= -\frac{8}{17}$ ✓ Answer

5.2 Express the following as a single trigonometric ratio. **The use of a calculator is not allowed**.

$$\frac{2}{\sin(30°+x)\cos x-\cos(30°+x)\sin x}\times\frac{\sin(138°)\cdot\sin(48°)}{1-2\sin^2 318°}$$

(7)

$$=\frac{2}{\sin(30°+x-x)}\times\frac{(\sin 42°)(\sin 48°)}{1-2(-\sin 42°)^2}$$

$$=\frac{2}{\sin 30°}\times\frac{\sin 42°\cdot\cos 42°}{1-2(\sin^2 42°)}$$

$$=\frac{\sin 84°}{\frac{1}{2}\cdot\cos 84°}$$

$$=2\tan 84°$$

✓ -sin 42°
✓ -sin 48°
✓ sin 30°
✓ cos 42°
✓ sin 84°
✓ $\frac{1}{2}$
✓ 2 tan 84°

OR

OR

$$=\frac{2}{\sin(30°+x-x)}\times\frac{(\sin 42°)(\sin 48°)}{\cos^2 318°}$$

$$=\frac{2}{\sin 30°}\times\frac{\sin 42°\cdot\cos 42°}{\cos 84°}$$

$$=\frac{\sin 84°}{\frac{1}{2}\cdot\cos 84°}$$

$$2\tan 84°$$

✓ -sin 42°
✓ -sin 48°
✓ cos 42°
✓ sin 84°
✓ $\frac{1}{2}$
✓ cos 84°
✓ 2 tan 84°

5.3 Given:

$$\frac{2\sin 2x-\cos^2 x-\sin^2 x+\cos 2x}{2\cos x-\sin x}=2\sin x$$

5.3.1 For which value(s) of x is the identity undefined? (4)

Identity is undefined if $2\cos x-\sin x=0$

$2\cos x=\sin x$

✓ $2\cos x-\sin x=0$

✓ $\tan x=2$

$2 = \frac{\sin x}{\cos x}$

$\tan x = 2$ ✓✓ Answer

$x = 63{,}43° + k \cdot 180°\ ;\ k\epsilon Z$

or

$x = 180° + 63{,}43° + k \cdot 180°\ ;\ k\epsilon Z$

$x = 243{,}43° + k \cdot 180°\ ;\ k \in Z$

5.3.2 Prove the identity completely. (5)

$LS = \frac{2(2\sin x\cos x) - (\cos^2 x + \sin^2 x) + (1 - 2\sin^2 x)}{2\cos x - \sin x}$

$= \frac{4\sin x\cos x - 1 + 1 - 2\sin^2 x}{2\cos x - \sin x}$

$= \frac{2\sin x\,(\cos x - \sin x)}{2\cos x - \sin x}$

$= 2\sin x$

$= RS$

✓ $2\ sin\ x\ cos\ x$
✓ $(\cos^2 x + \sin^2 x)$
✓ $1 - 2\sin^2 x$
✓ 1
✓ $2\sin x(\cos x - \sin x)$

OR

OR

$LS = \frac{2(2\sin x\cos x) - \cos^2 x - \sin^2 x + (\cos^2 x + \sin^2 x) - 2\sin^2 x}{2\cos x - \sin x}$

$= \frac{4\sin x\cos x - 2\sin^2 x}{2\cos x - \sin x}$

$= \frac{2\sin x\,(\cos x - \sin x)}{2\cos x - \sin x}$

$= 2\sin x$

$= RS$

✓ $(\cos^2 x + \sin^2 x)$
✓ $4\ sin\ x\cos x$
✓ 4sin $x$cosx - 2sin^2x
✓ $2\sin x$
✓ $(\cos x - \sin x)$

OR

OR

$LS = \frac{2(2\sin x\cos x) - \cos^2 x - \sin^2 x + (2\cos^2 x - 1)}{2\cos x - \sin x}$

$= \frac{4\sin x\cos x - \cos^2 x - \sin^2 x + 2\cos^2 x - (\cos^2 x + \sin^2 x)}{2\cos x - \sin x}$

$= \frac{2\sin x\,(\cos x - \sin x)}{2\cos x - \sin x}$

✓ $2\cos^2 x - 1$
✓ $4\ sin\ x\cos x$
✓ $(\cos^2 x + \sin^2 x)$
✓ $2\sin x$
✓ $(\cos x - \sin x)$

$= 2\sin x$

$= RS$

5.3.3 Hence, or otherwise, determine the value(s) of x such that

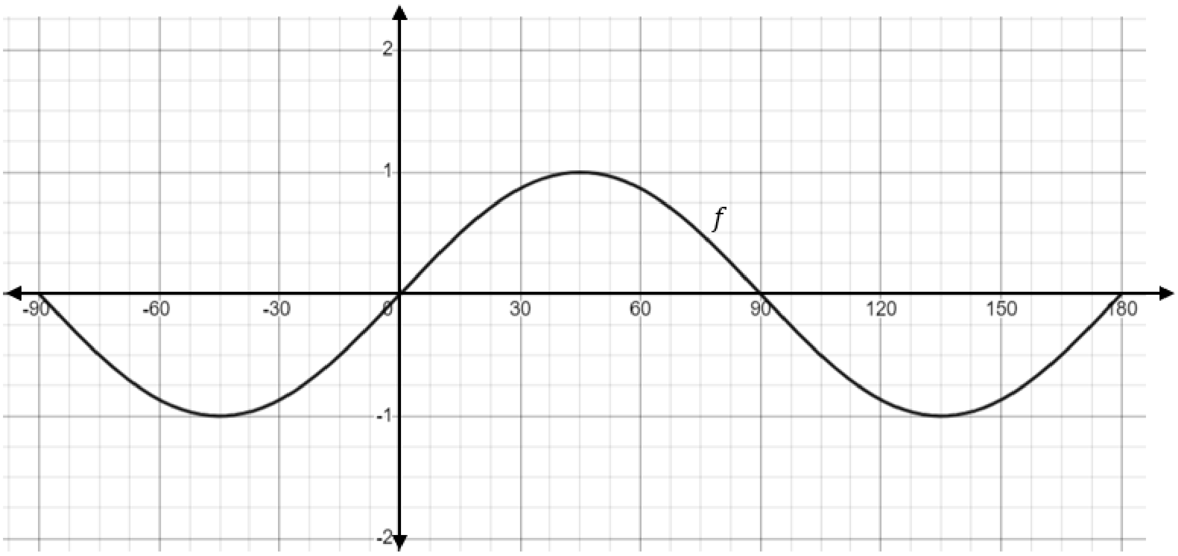
$2\sin 2x - \cos^2 x - \sin^2 x + \cos 2x = 4\cos x - 2\sin x\,, \quad if\ x\epsilon[-360°; 360°]$ (4)

$2\sin 2x - \cos^2 x - \sin^2 x + \cos 2x = 2(2\cos x - \sin x)$	✓ Common factor $2(2\cos x - \sin x)$
$\dfrac{2\sin 2x - \cos^2 x - \sin^2 x + \cos 2x}{2\cos x - \sin x} = 2$	✓ ÷ both sides $2\cos x - \sin x$
$\therefore 2\sin x = 2$ $\sin x = 1$	✓ 2 sin x=2
$x\epsilon\{-270°\,; 90°\}$	✓ -270° AND 90°

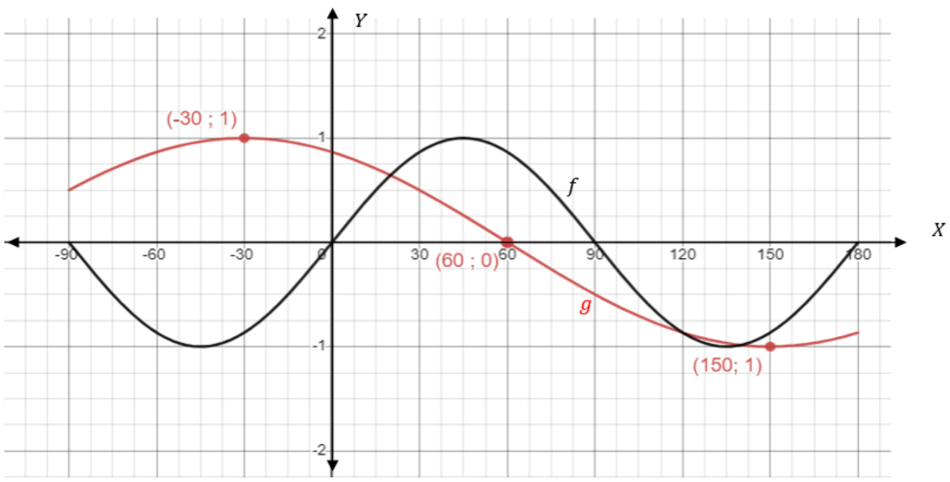
[29]

QUESTION 6

The diagram below shows the graph of $f(x) = \sin 2x$, where $x\epsilon[-90°; 180°]$.



6.1 Sketch the graph of $g(x) = \cos(x + 30°)$ **on the same system of axes above** for $x\epsilon[-90°; 180°]$. (4)



✓ (-30°;1)

✓ (60°;0)

✓ (150°; -1)

✓ shape

6.2 Determine the x-value(s) for the points of intersection of f and g, if $x\epsilon[-90°; 180°]$. (4)

$f(x) = g(x)$
$\sin 2x = \cos(x + 30°)$
$\sin 2x = \sin[90° - (x + 30°)]$ ✓ $\sin(60° - x)$

R.A= $60° - x$
$2x = 60° - x + k.360°\,; k\epsilon Z$ or $2x = 180 - (60° - x) + k.360°\,; k\epsilon Z$
$3x = 60° + k.360°$ $x = 120° + k.360°$
$x = 20° + k.120°;\ k\epsilon Z$ ✓ $20° + k.120°$
$x\epsilon\{20°\,; 120°; 140°\}$ ✓✓ All THREE x –values

6.3 Use your graph and the value(s) calculated in 6.1 and determine the value(s) of x such that:

6.3.1 $f'(x) = 0$, if $x > 0$ (2)

$x = 45°$ or $x = 135°$

✓ $x = 45°$
✓ $x = 135°$

6.3.2 $\sin x \cos x > \frac{1}{2}\cos(x + 30°)$ (5)

CA from 6.1

$2 \sin x \cos x > \cos(x + 30°)$

$\sin 2x > \cos(x + 30°)$

$x \epsilon (20° ; 120°) \cup (140° ; 180°]$

✓$\sin 2x > \cos(x + 30)$
✓ (20°; ✓ 120°)

OR

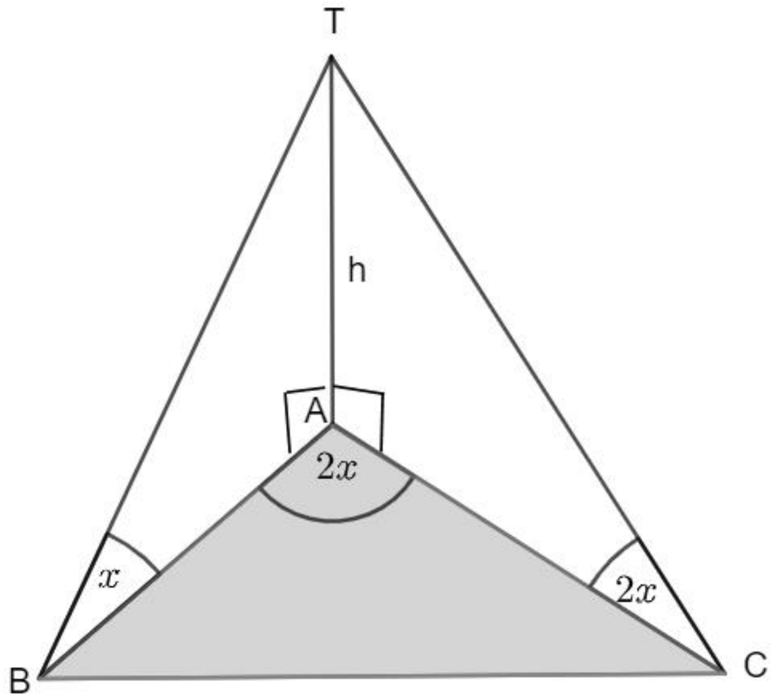
$20° < x < 120°$ or $140° < x \leq 180°$

✓ (140°
✓ 180°]

[15]

QUESTION 7

In the diagram AT is a vertical tower.
A, B and C lie in the same horizontal plane. The angle of elevation to the top of the tower from B is x and the angle of elevation to the top of the tower from C is $2x$.
$B\hat{A}C = 2x$.



7.1 Express AC and AB in terms of x and h. (3)

$\tan A\hat{C}T = \dfrac{AT}{AC}$	✓ Tan ratio
$AC = \dfrac{h}{\tan 2x}$	✓ $AC = \dfrac{h}{\tan 2x}$
$\tan A\hat{B}T = \dfrac{AT}{AB}$	
$AB = \dfrac{h}{\tan x}$	✓ $AB = \dfrac{h}{\tan x}$

7.2 Hence, prove that $Area\ \Delta ABC = \dfrac{h^2 \cos 2x}{2 \tan x}$ (4)

$$Area\ \Delta ABC = \frac{1}{2} \cdot AB \cdot AC \sin B\hat{A}C$$

$$Area\ \Delta ABC = \frac{1}{2}\left(\frac{h}{\tan x}\right)\left(\frac{h}{\tan 2x}\right) \sin 2x$$

✓ Corr subst in corr formula

$$Area\ \Delta ABC = \frac{h^2 \sin 2x}{2 \tan x \cdot \frac{\sin 2x}{\cos 2x}}$$

✓ tan 2x ident.

✓ x reciprocal

$$Area\ \Delta ABC = \frac{h^2 \sin 2x}{2 \tan x} \times \frac{\cos 2x}{\sin 2x}$$

$$Area\ \Delta ABC = \frac{h^2 \cos 2x}{2 \tan x}$$

7.3 Determine the area of ΔABC if $h = 12$ and $x = 32°$. (2)

$$Area\ \Delta ABC = \frac{(12)^2 \cos 2(32)}{2 \tan(32)}$$

✓ Subst.

$$Area\ \Delta ABC = 50{,}51\ units^2$$

✓ Answer

[9]

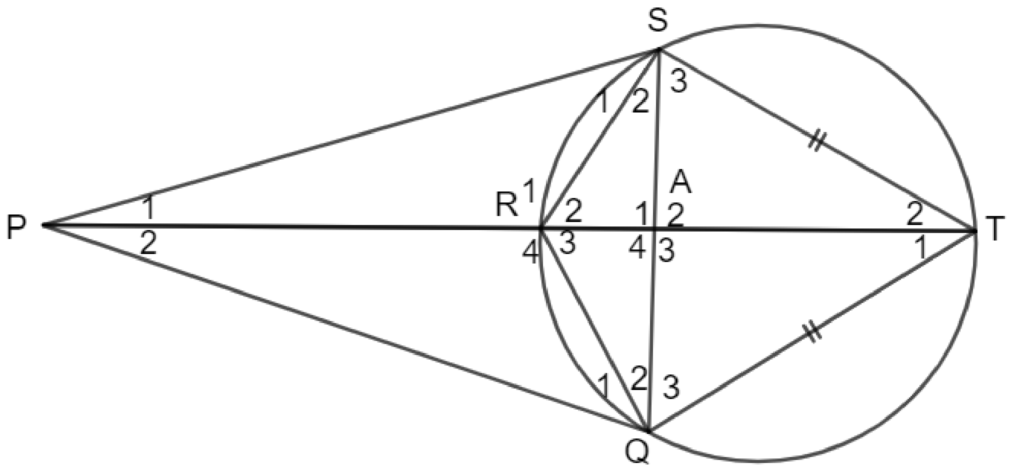
QUESTION 8

8.1 Complete the following statement correctly:
The angle between a tangent to a circle and a chord drawn from the point of contact is . . . (2)

. . . equal to an angle in the alternate circle segment of the circle. ✓✓ Answer

8.2 In the diagram:

- Q, R, S and T lie on the circumference of the circle.
- ST = QT.
- PS and PQ are tangents to the circle at S and Q respectively.
- PRAT is a straight line and intersects straight line SQ in A.



8.2.1 Give a reason why $P\hat{S}Q = P\hat{Q}S$. (2)

$PS = PQ$ ✓✓ R

$\therefore P\hat{S}Q = P\hat{Q}S$ . . . ∠'s opp = sides

OR

$P\hat{S}Q = P\hat{Q}S$ . . . Tangents from common point outside circle.

8.2.2 Prove that $Q\hat{R}S = 2\hat{S}_3$. (3)

$\hat{S}_3 = \hat{Q}_3$ . . . ∠'s opposite = sides ✓ S & R

$\hat{R}_3 = \hat{S}_3$ . . . ∠'s in same segment ✓ S & R

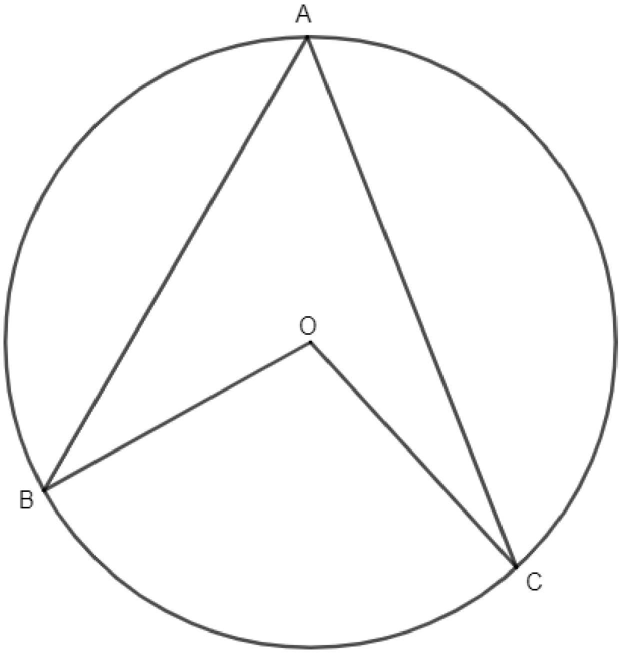
$\hat{Q}_3 = \hat{R}_2$ . . . ∠'s in same segment ✓ S & R

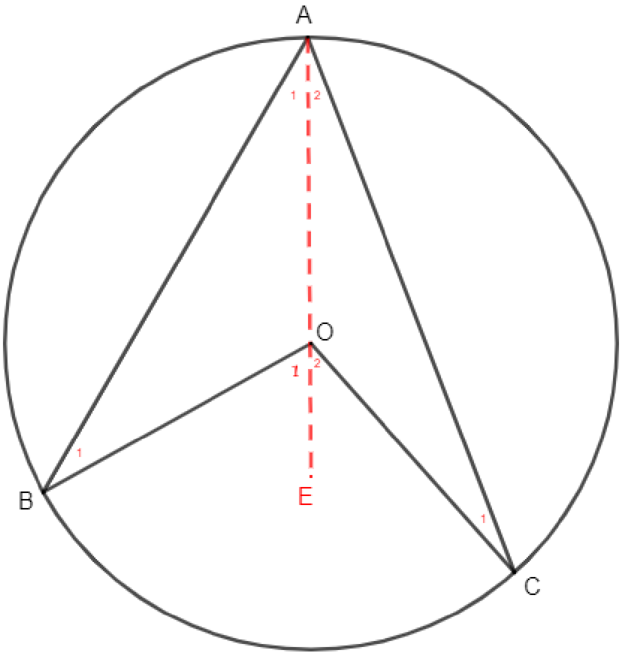
$Q\hat{R}S = \hat{Q}_3 + \hat{Q}_3 = 2\hat{S}_3$ (3)

[7]

QUESTION 9

9.1 Use the given diagram to prove that $B\hat{O}C = 2B\hat{A}C$. (5)





Construction: Draw AO and extend to D: ✓ Construction

$\hat{A}_1 = \hat{B}$ . . . ∠'s opposite = sides AO=OB (radii)
And $\hat{A}_2 = \hat{C}$ . . . . ∠'s opposite = sides AO=OC (radii)

✓ ∠'s opposite = sides

$\hat{O}_1 = \hat{A}_1 + \hat{B}$ . . . ext ∠ = sum opp int ∠'s of △AOB
$\therefore\ \hat{O}_1 = 2A_1$

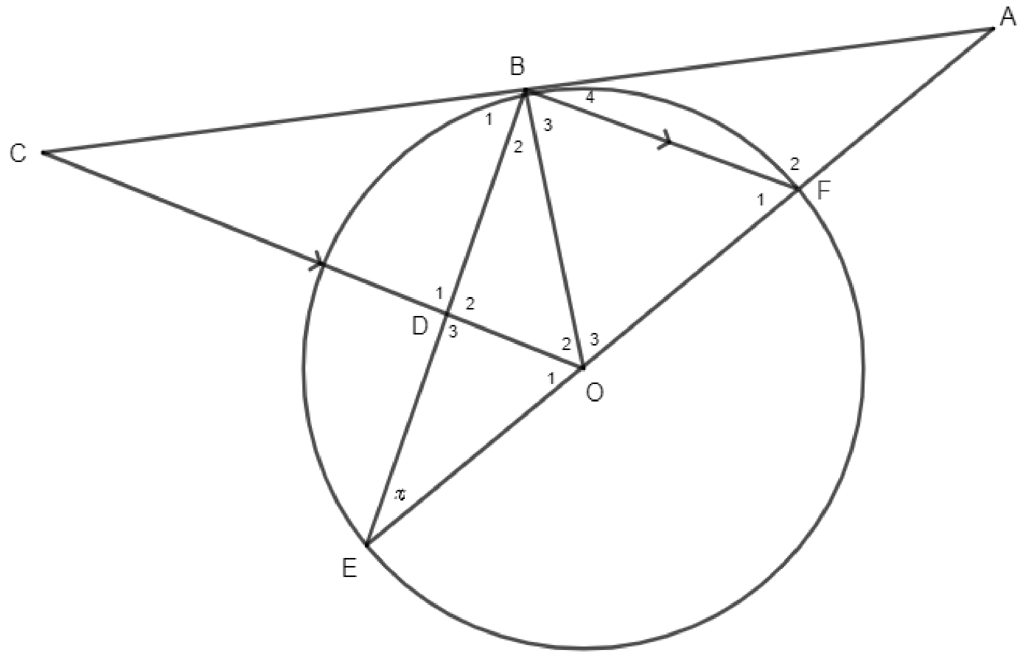
✓ ext ∠ = sum opp

$\hat{O}_2 = \hat{A}_2 + \hat{C}$. . . ext ∠ = sum opp int ∠'s of △AOC
∴ $\hat{O}_2 = 2\hat{A}_2$

$B\hat{O}C = \hat{O}_1 + \hat{O}_2$ ✓ $B\hat{O}C = \hat{O}_1 + \hat{O}_2$
$B\hat{O}C = 2\hat{A}_1 + 2A_2$
$B\hat{O}C = 2(\hat{A}_1 + \hat{A}_2)$
$B\hat{O}C = 2B\hat{A}C$ ✓ $B\hat{O}C = 2(\hat{A}_1 + \hat{A}_2)$

9.2 In the diagram:

- O is the centre of the circle.
- EF, the diameter of the circle is extended to A.
- ABC is a tangent to the circle at B.
- CO ∥ BF.
- $\hat{E} = x$.



9.2.1 State, with reasons, THREE other angles equal to x. (3)

$\hat{B}_4 = x$ . . . . tan-chord ✓ S&R
$\hat{B}_2 = x$ . . . .∠'s opp = sides (BO=EO radii) ✓ S&R
$\hat{C} = \hat{B}_4 = x$ . . . .corresponing $\angle's$, $BF \parallel CO$ ✓ S&R

9.2.2 Determine, with reasons, $\hat{O}_3$ in terms of x. (2)

$\hat{O}_3 = 2x$ . . . ∠ at centre = 2x ∠ at circumference ✓ $2x$ ✓ R

OR

$\hat{O}_3 = 2x$ . . . Ext ∠ = sum opp int ∠'s

9.2.3 Prove that D is the midpoint of BE. (4)

$E\hat{B}F = 90°$ . . . ∠ in semi-circle ✓ S ✓ R

∴ $\hat{D}_2 = 90°$ . . . Co-Int ∠'s; BF||CO ✓ S & R

∴ $OD \perp BE$ ✓ R

∴ $BD = DE$ . . . line from midpoint ⊥ on chord

.2.4 Prove that BOEC is a cyclic quadrilateral. (2)

$\hat{C} = \hat{E} = x$ . . . proven ✓ $\hat{C} = \hat{E} = x$
BOEC is cyclic quadr. . . . converse ∠'s in same circlesegment ✓ R

9.2.5 Prove that ΔABF ||| ΔAEB. (3)

In ΔABF and ΔAEB:
$\hat{A} = \hat{A}$ . . . common ✓ Common ∠
$\hat{B}_4 = \hat{E} = x$ . . . proven ✓ 2nd ∠
∴ $A\hat{F}B = A\hat{B}E$ . . . int ∠'s of Δ's ✓ R

∴ $\Delta ABF \parallel\!\mid \Delta AEB$ . . . A,A,A

9.2.6 Prove that $2ED.AB = AE.BF$. (3)

$\frac{EB}{BF} = \frac{AE}{AB}$ . . . $\Delta ABF \parallel\!\mid \Delta AEB$ ✓ Ratio

$EB\,.AB = BF\,.AE$ ✓ Simplify ✓ $EB = 2ED$

But $EB = 2.ED$ . . . D is midpoint of EB

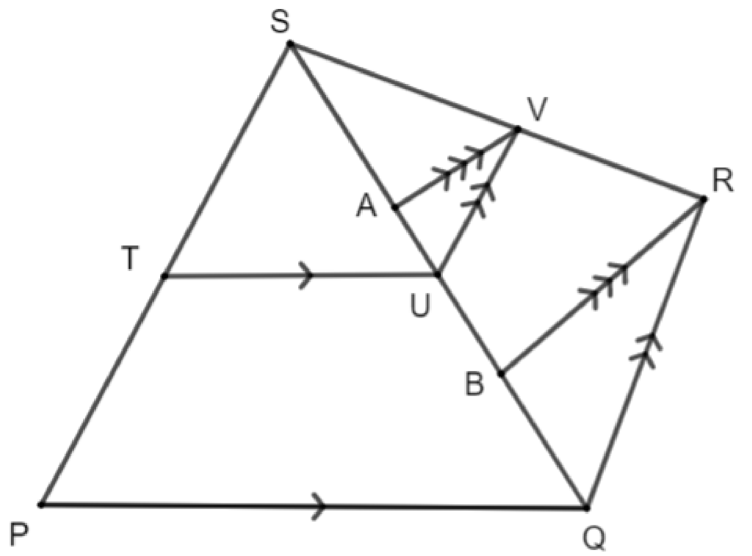
∴ $2ED.AB = BF.AE$

[22]

QUESTION 10

In the diagram ΔSPQ and ΔSQR are given:

- T lies on SP and U on SQ with TU || PQ.
- V lies on SR with UV || QR.
- A and B lie on SQ with AV || BR.
- SP = 14 units.
- ST = 8 units.
- B is the midpoint of UQ.



10.1 Determine, with reasons, the ratio of SV : VR. (4)

$TP = 14 - 8 = 6$ units ✓ TP=6

$\frac{SV}{VR} = \frac{SU}{UQ}$ ...prop theorem, $UV \parallel QR$ **OR** line $\parallel$ one side of Δ ✓ S & R

$\frac{SU}{UQ} = \frac{ST}{TP}$...prop theorem $TU \parallel PQ$ **OR** line $\parallel$ one side of Δ ✓ S & R

$\therefore \frac{SV}{VR} = \frac{ST}{TP} = \frac{8}{6} = \frac{4}{3}$ ✓ $\frac{4}{3}$

10.2 Determine, with reasons, the ratio of BR : AV. (3)

$\therefore \frac{RB}{VA} = \frac{SR}{SV}$ ...$\Delta SVA ||| \Delta SRB$ **OR** prop theorem; AV‖BR **OR** line ‖ one side of Δ ✓ S & R

✓ $SV + VR$

$\frac{RB}{VA} = \frac{SV + VR}{SV} = \frac{8+6}{8} = \frac{14}{8} = \frac{7}{4}$ ✓ $\frac{7}{4}$

10.3 Show that $8UB = 3SU$. (3)

$\frac{SU}{UQ} = \frac{SV}{VR} = \frac{4}{3}$ ...prop theorem, $UV \parallel QR$ ***OR*** line ‖ one side of Δ ✓ S & R

$UB = BQ$ . . . given B midpoint of UQ ✓ $UB = BQ$

$\therefore \frac{SU}{UB} = \frac{4}{1,5} = \frac{8}{3}$ ✓ $\frac{SU}{UB} = \frac{8}{3}$

$3SU = 8UB$

[10]

GRAND TOTAL: [150]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$