EXAMINATION	NATIONAL SENIOR CERTIFICATE
GRADE	12
DATE	JUNE 2024
SUBJECT	MATHEMATICS
PAPER	2
MARK TOTAL	150
DURATION (HOURS)	3
NUMBER OF PAGES	27

SOUTH AFRICAN COMPREHENSIVE ASSESSMENT INSTITUTE

SUID-AFRIKAANSE KOMPREHENSIEWE ASSESSERINGSINSTITUUT

QUESTION 1

To promote blood donation a blood donor centre was established at a shopping centre. The table provided below displays the daily count of blood units donated in TEN days by the people at the shopping centre.

Days	1	2	3	4	5	6	7	8	9	10
Units of blood	45	59	65	73	79	85	91	99	101	110

1.1.1 Calculate the mean of the units of blood donated per day over the period of 10 days. (2)

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1.1.2 Determine the standard deviation of the data. (2)

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1.1.3 How many days was the number of units of blood donated at the shopping centre outside one standard deviation from the mean? (3)

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1.2.1 Draw a box and whisker diagram to represent the number of units of blood donated by people at the shopping centre. (3)

1.2.2 Describe the skewness of the data. Motivate your answer. (2)

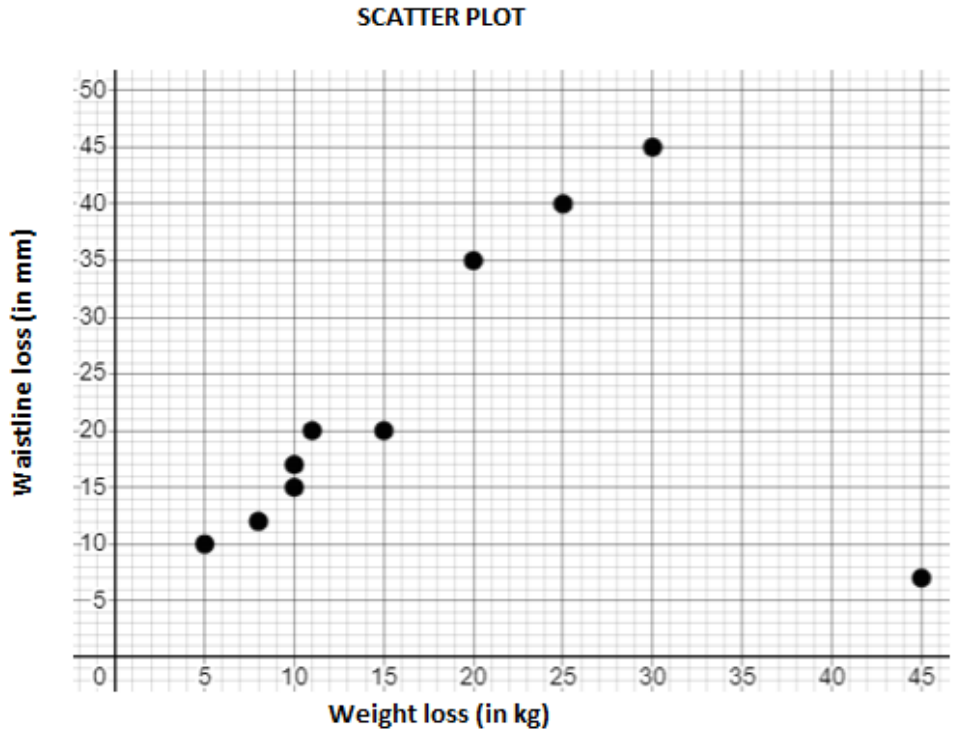
[12]

QUESTION 2

The table below represents the weight loss and waistline loss of 10 contestants who took part in a weight loss competition.

Weight loss (in kg) (x)	**5**	**10**	**45**	**8**	**10**	**15**	**25**	**30**	**11**	**20**
Waistline loss (in mm) (y)	**10**	**15**	**7**	**12**	**17**	**20**	**40**	**45**	**20**	**35**

A scatter plot of the above results is shown below.



2.1 Identify an outlier in the data above. (1)

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2.2 Determine the equation of the least squares regression line for the data. (2)

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2.3 If a contestant lost 21,2 kg during the same time. Predict the waistline loss of this contestant, correct to two decimal places. (2)

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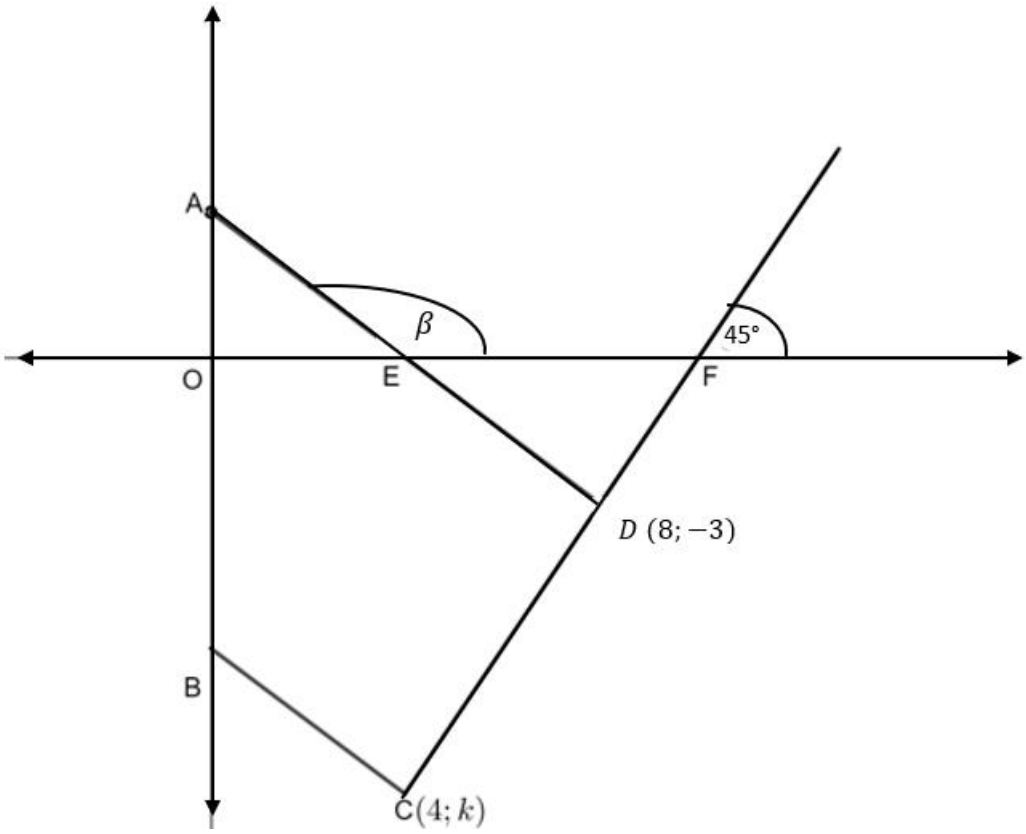
2.4 Draw the least square regression line on the scatter plot **provided above**. (3)

[8]

QUESTION 3

In the diagram below:

- AD is a straight line with the equation $y = -\frac{3}{4}x + 3$.
- E and A are the $x-$ and $y-$intercepts respectively of AD .
- The angle of inclination of AD is β.
- From $C(4; k)$ a line, parallel to AD is drawn that intersects the $y-$axes at B.
- Line CD is drawn through $D(8; -3)$ and intersects the x-axis at F.
- The angle of inclination of CD is $45°$.



3.1 Determine the coordinates of A and E. (4)

3.2 Show that $k = -7$. (3)

3.3 Determine the equation of BC. (3)

3.4 Determine whether ΔDEF is right-angled. Show all your calculations. (4)

3.5 Calculate the area of $\triangle DEF$. (5)

3.6 Let G be a point in the fourth quadrant such that CEDG forms a parallelogram. Calculate the coordinates of G. (4)

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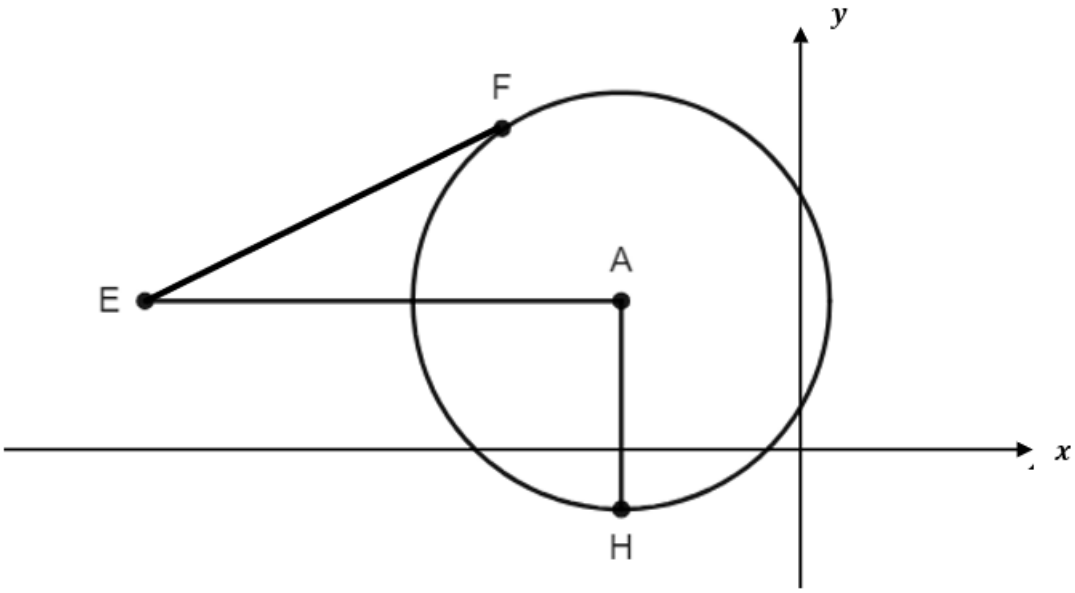
QUESTION 4

In the diagram circle with centre A and equation $x^2 + y^2 + 12x - 10y = -12$ is given.
AE is parallel to the x-axes and AH is perpendicular to the x −axes.
EF is a tangent to the circle such that $EF = \sqrt{95}$.



4.1 Determine the coordinates of A and the radius of the circle. (5)

4.2 Determine the equation of the tangent to the circle at point H. (1)

4.3 Determine the coordinates of E. (3)

4.4 Calculate the size of $F\hat{E}A$. (2)

4.5 A circle with centre B and represented by the equation $(x + 10)^2 + (y - 5)^2 = k$, touches circle A internally.
Calculate the value of k. (2)

4.6 Circle A is reflected about the *x*-axis and translated 2 units to the right to form a new circle with centre P.
Write down the equation of circle P. (2)

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[15]

QUESTION 5

5.1 If $17\cos\theta = -8$ and $0° < \theta < 180°$, calculate, **without the use of a calculator**:

5.1.1 $\tan(360° - \theta)$ (3)

5.1.2 $\sin 2\theta$. (3)

5.1.3 $\sin(810° - \theta)$. (3)

5.2 Express the following as a single trigonometric ratio. **The use of a calculator is not allowed**.

$$\frac{2}{\sin(30° + x)\cos x - \cos(30° + x)\sin x} \times \frac{\sin(138°) \cdot \sin(48°)}{1 - 2\sin^2 318°} \quad (7)$$

5.3 Given:

$$\frac{2\sin 2x - \cos^2 x - \sin^2 x + \cos 2x}{2\cos x - \sin x} = 2\sin x$$

5.3.1 For which value(s) of x is the identity undefined? (4)

5.3.2 Prove the identity completely. (5)

5.3.3 Hence, or otherwise, determine the value(s) of x such that

$2\sin 2x - \cos^2 x - \sin^2 x + \cos 2x = 4\cos x - 2\sin x\,, \quad if\ x\epsilon[-360°; 360°]$ (4)

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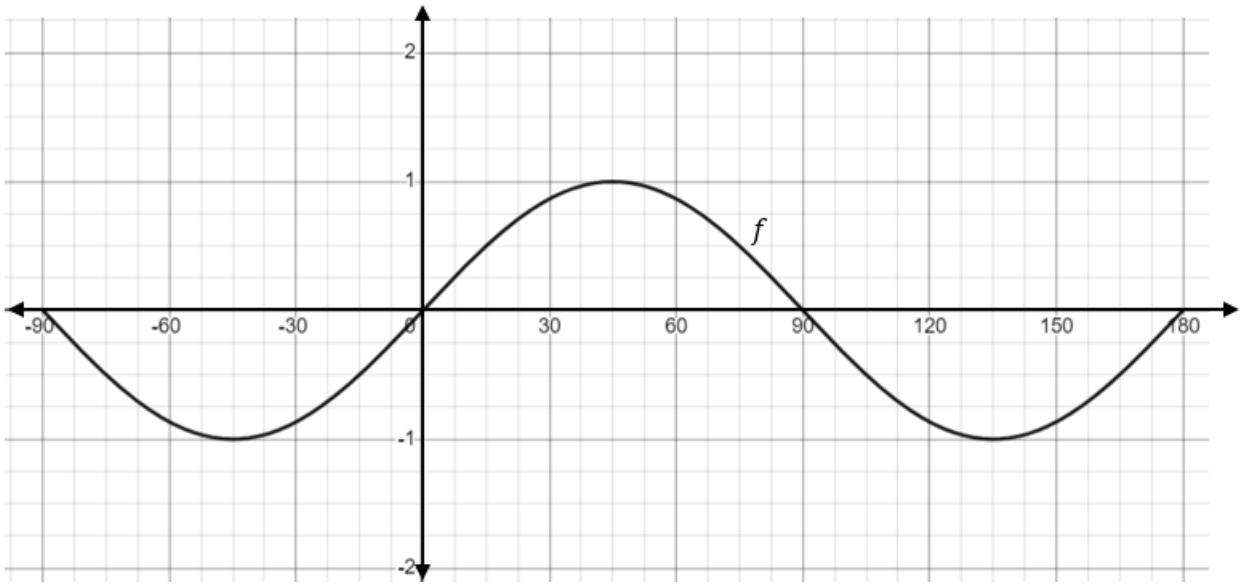
QUESTION 6

The diagram below shows the graph of $f(x) = \sin 2x$, where $x \epsilon [-90°; 180°]$.



6.1 Sketch the graph of $g(x) = \cos(x + 30°)$ **on the same system of axes above** for $x \epsilon [-90°; 180°]$. (4)

6.2 Determine the x-value(s) for the points of intersection of f and g, if $x \epsilon [-90°; 180°]$. (4)

6.3 Use your graph and the value(s) calculated in 6.1 and determine the value(s) of x such that:

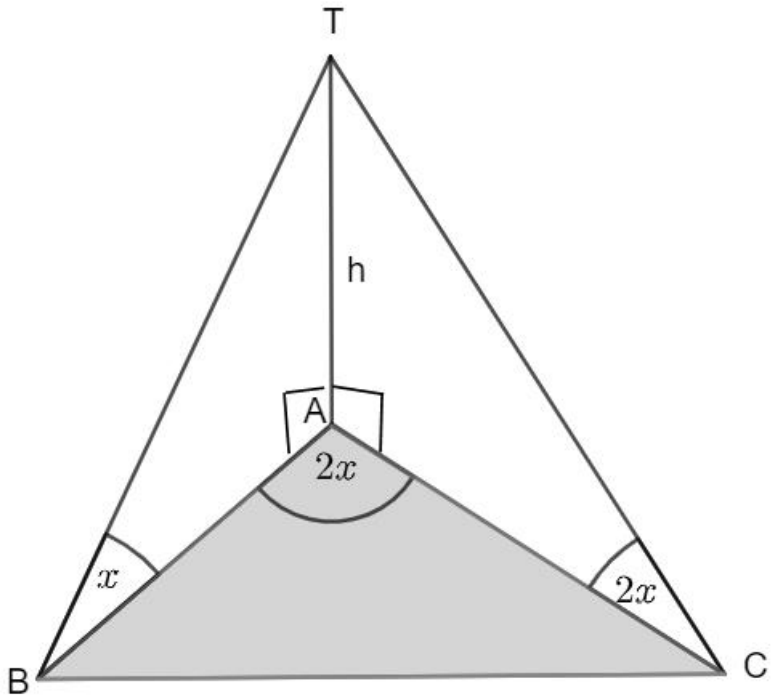
6.3.1 $f'(x) = 0$, if $x > 0$ (2)

6.3.2 $\sin x \cos x > \frac{1}{2}\cos(x + 30°)$ (5)

[15]

QUESTION 7

In the diagram AT is a vertical tower.
A, B and C lie in the same horizontal plane. The angle of elevation to the top of the tower from B is x and the angle of elevation to the top of the tower from C is $2x$.
$B\hat{A}C = 2x$.



7.1 Express AC and AB in terms of x and h. (3)

7.2 Hence, prove that $Area\ \Delta ABC = \frac{h^2 \cos 2x}{2 \tan x}$ (4)

7.3 Determine the area of ΔABC if $h = 12$ and $x = 32°$. (2)

[9]

QUESTION 8

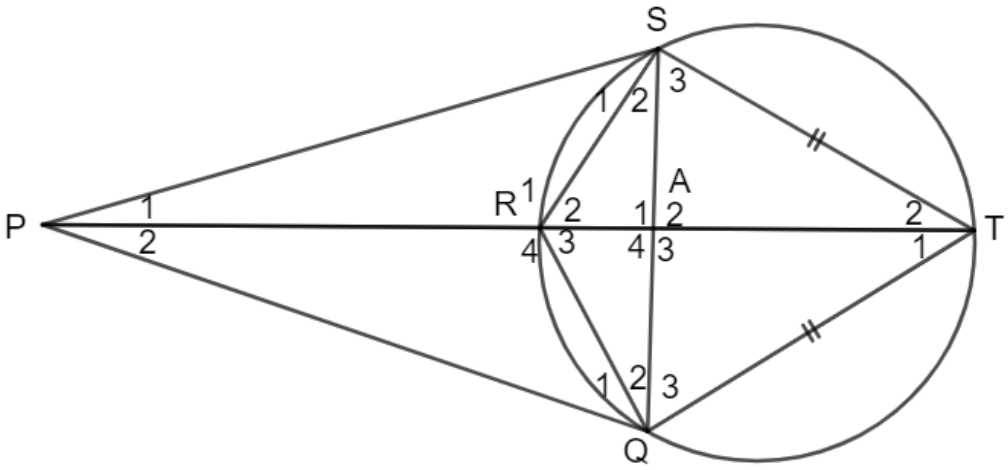
8.1 Complete the following statement correctly:
The angle between a tangent to a circle and a chord drawn from the point of contact is . . . (2)

_____

_____

8.2 In the diagram:

- Q, R, S and T lie on the circumference of the circle.
- ST = QT.
- PS and PQ are tangents to the circle at S and Q respectively.
- PRAT is a straight line and intersects straight line SQ in A.



8.2.1 Give a reason why $P\hat{S}Q = P\hat{Q}S$. (2)

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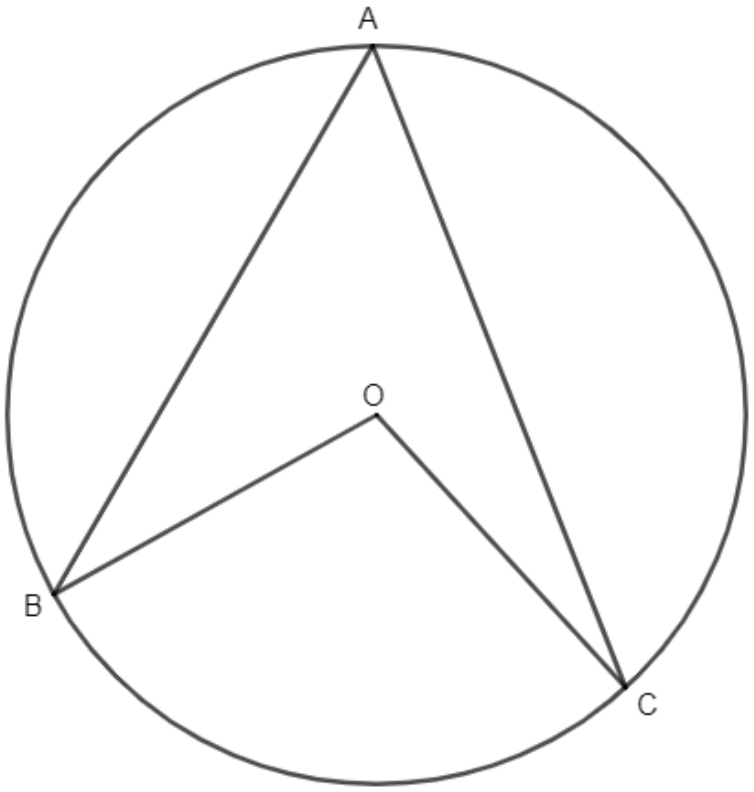
8.2.2 Prove that $Q\hat{R}S = 2\hat{S}_3$. (3)

[7]

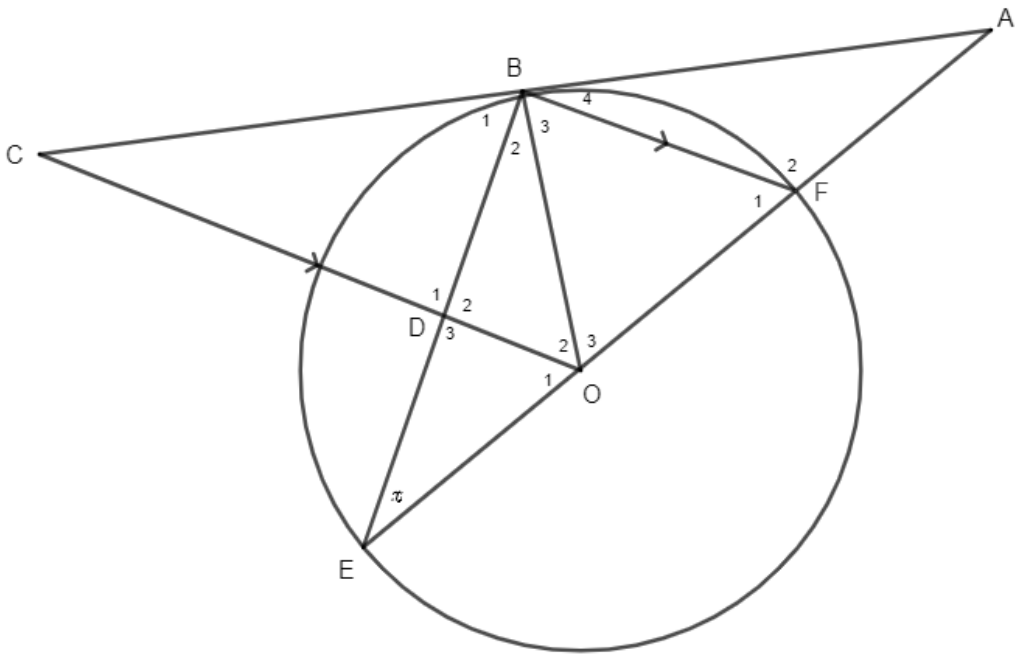
QUESTION 9

9.1 Use the given diagram to prove that $B\hat{O}C = 2B\hat{A}C$. (5)



9.2 In the diagram:

- O is the centre of the circle.
- EF, the diameter of the circle is extended to A.
- ABC is a tangent to the circle at B.
- CO ∥ BF.
- $\hat{E} = x$.



9.2.1 State, with reasons, THREE other angles equal to x. (3)

9.2.2 Determine, with reasons, $\hat{O}_3$ in terms of x. (2)

9.2.3 Prove that D is the midpoint of BE. (4)

9.2.4 Prove that BOEC is a cyclic quadrilateral. (2)

9.2.5 Prove that ΔABF ||| ΔAEB. (3)

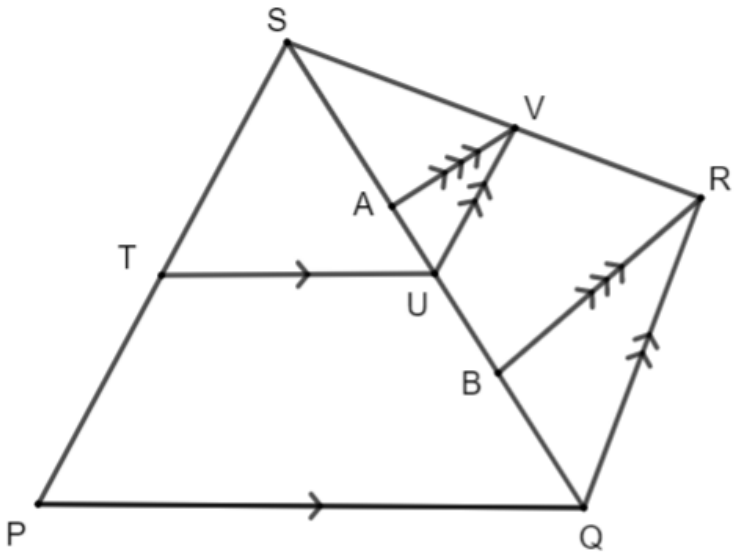
9.2.6 Prove that $2ED.AB = AE.BF$. (3)

[22]

QUESTION 10

In the diagram ΔSPQ and ΔSQR are given:

- T lies on SP and U on SQ with TU || PQ.
- V lies on SR with UV || QR.
- A and B lie on SQ with AV || BR.
- SP = 14 units.
- ST = 8 units.
- B is the midpoint of UQ.



10.1 Determine, with reasons, the ratio of SV : VR. (4)

10.2 Determine, with reasons, the ratio of BR : AV. (3)

10.3 Show that $8UB = 3SU$. (3)

[10]

GRAND TOTAL: [150]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$T_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d]$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a(r^n - 1)}{r-1} \; ; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \; ; \; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \text{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$

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