EXAMINATION	NATIONAL SENIOR CERTIFICATE
GRADE	12
DATE	MAY/JUNE 2025
SUBJECT	MATHEMATICS
PAPER	2
MARK TOTAL	150
DURATION (HOURS)	3
NUMBER OF PAGES	30

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SOUTH AFRICAN COMPREHENSIVE ASSESSMENT INSTITUTE

SUID-AFRIKAANSE KOMPREHENSIEWE ASSESSERINGSINSTITUUT

INSTRUCTIONS

Read the following instructions carefully before answering the questions.

1. This paper consists of 10 questions. Answer all the questions.
2. Clearly show **ALL** calculations, diagrams, graphs etc. that you have used in determining your answers.
3. Answers only will not necessarily be rewarded full marks.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round off answers to **TWO** decimal places, unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. An information sheet with formulae is included at the end of the question paper.
8. Write neatly and legibly, in **BLUE** ink only.
9. Answer all the questions **on the exam paper** on the lines provided after each question.
10. Additional writing space is provided at the end of the paper. Clearly indicate if you use the additional writing space to complete a question.

QUESTION 1

In a survey, 80 learners from Ntsako High School were asked how many hours they spend, per week, watching their favourite television shows. The frequency table below represents the distribution of the amount of time spent watching their favourite television shows.

Time in hours	Frequency
$10 < t \leq 15$	8
$15 < t \leq 20$	28
$20 < t \leq 25$	27
$25 < t \leq 30$	12
$30 < t \leq 35$	4
$35 < t \leq 40$	1

1.1 State the modal interval of the data. (1)

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1.2 Draw an ogive (cumulative frequency graph) to illustrate the data on the grid provided below. (4)

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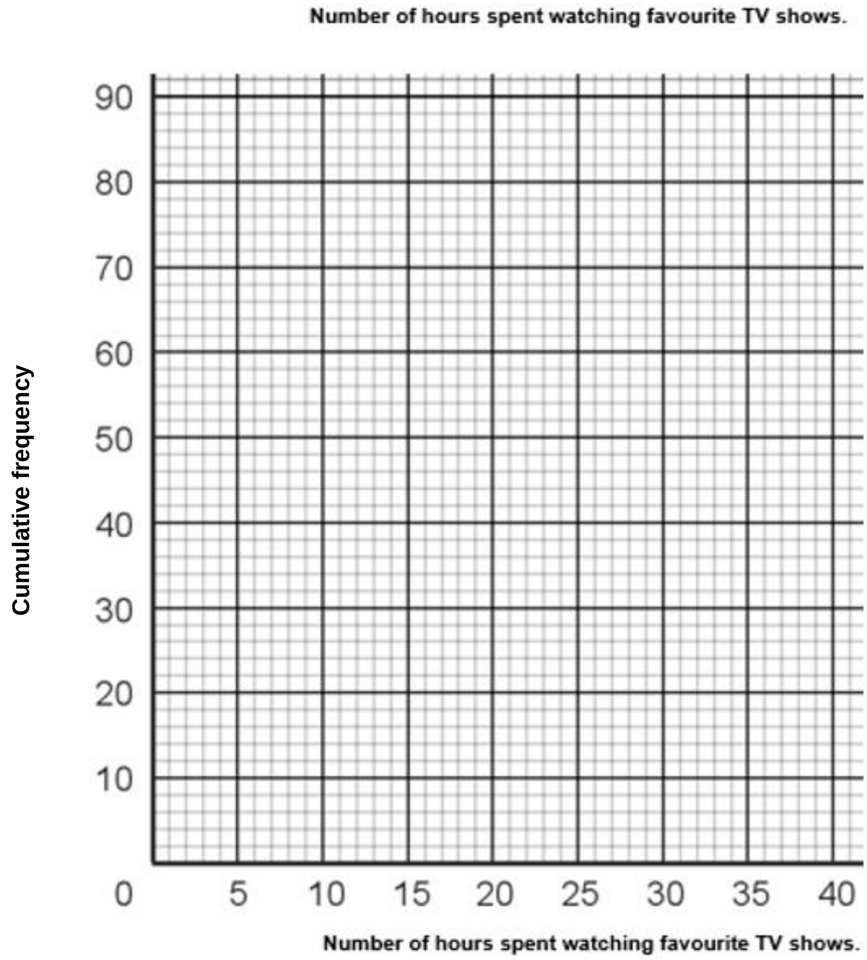
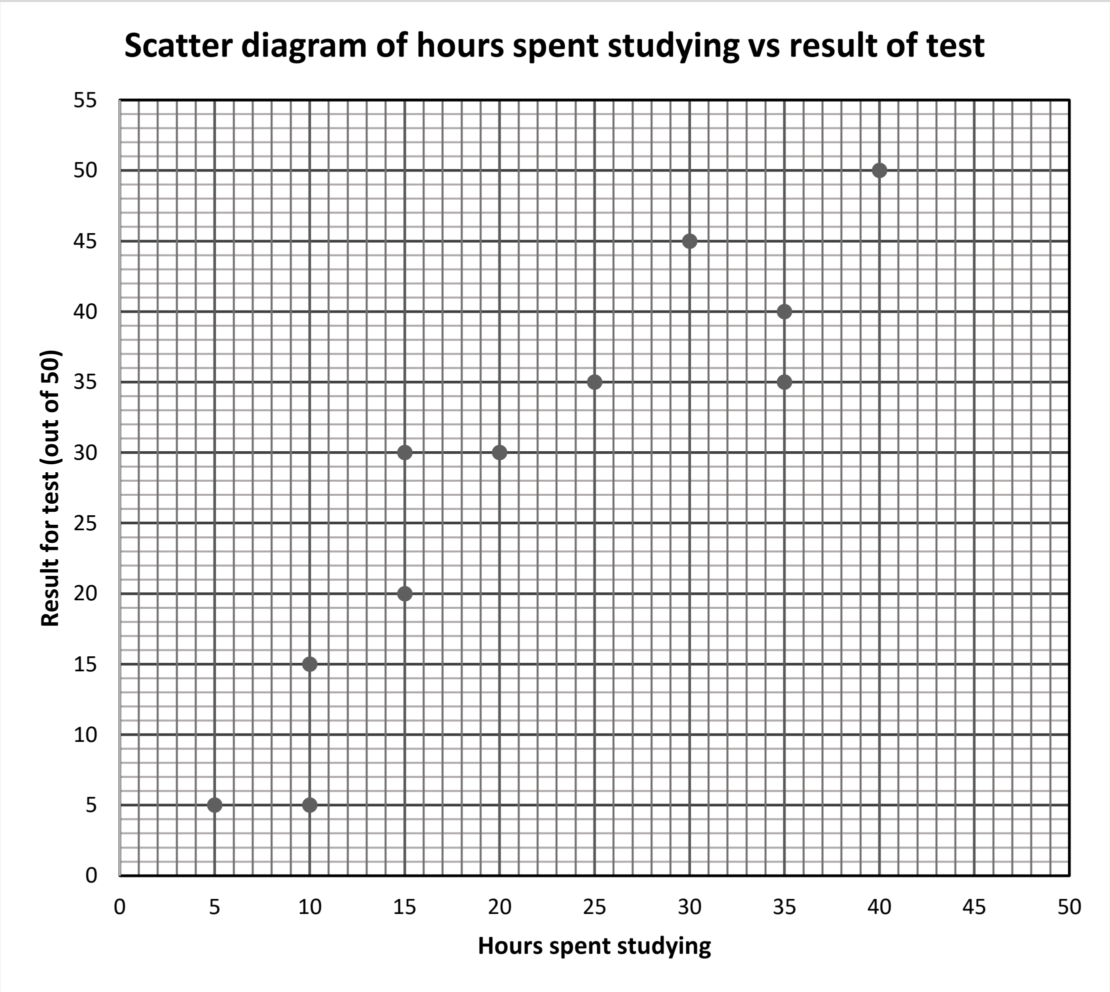


1.3 **Use your graph** to determine an estimate of the median of the data. Indicate your answer by placing a capital **M** on your graph. (2)

[7]

QUESTION 2

2.1 **Eleven** grade 12 learners are interested in understanding how the amount of time they spend studying correlates with their test scores. Each learner records the number of hours they spend studying for Mathematics in a week, and their corresponding test score out of 50. They present their data on the scatter plot diagram below.



2.1.1 Calculate the equation of the least squares regression line of the data. (3)

2.1.2 Draw the least squares regression line on the scatter plot provided in the question. Indicate the average for both the study time and the result for the test on the diagram. (2)

2.1.3 Predict, to the nearest whole number, the test score of a learner who spends 18,5 hours studying for the Mathematics test. (2)

2.1.4 Determine the correlation coefficient of the data and comment on the correlation between the study time and the test result. (2)

2.2 The data below represents the marks achieved by seven Grade 12 learners for their Mathematics test (out of 20) in ascending order. None of the marks are repeated.

Mathematics marks (20)	8	a	12	b	c	d	20

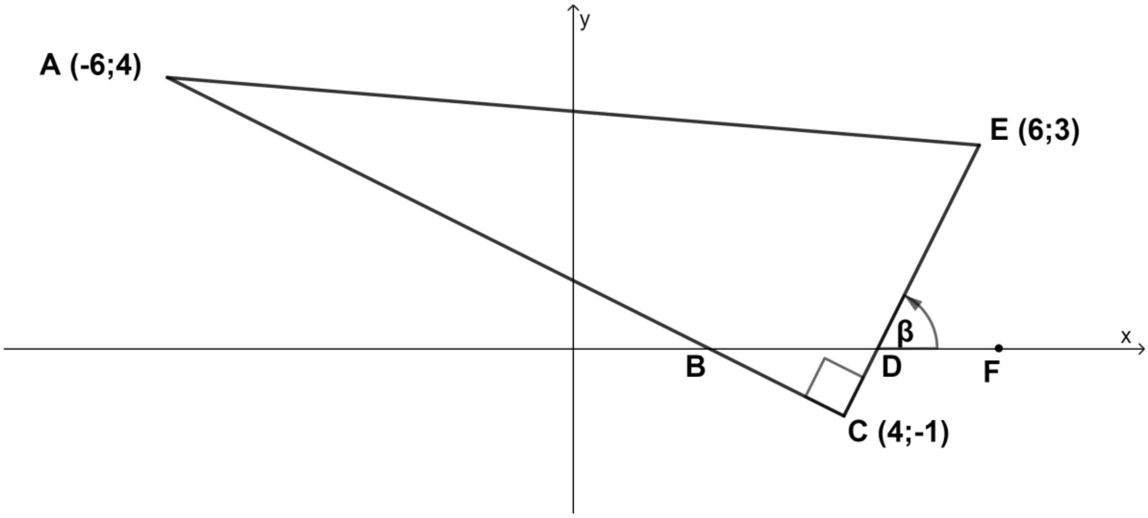
Determine the values of a, b, c and d if:

- the median is 14.
- the semi-interquartile range is 4,5.
- the upper quartile is twice the lower quartile.
- the mean is 14. (6)

[15]

QUESTION 3

In the diagram below $A\,(-6;4), C(4;-1)$ and $E(6;3)$ are the vertices of a triangle with $\hat{C} = 90°$. AC intersects the x-axis at B and EC intersects the x-axis at D. F is a point on the positive x-axis. $E\hat{D}F = \beta$.



3.1 Calculate the gradient of EC. (2)

3.2 Determine the equation of the line PA, which passes through A and is parallel to EC. (3)

3.3 Calculate the area of ΔACE. (5)

3.4 Calculate the size of $D\hat{B}C$. (4)

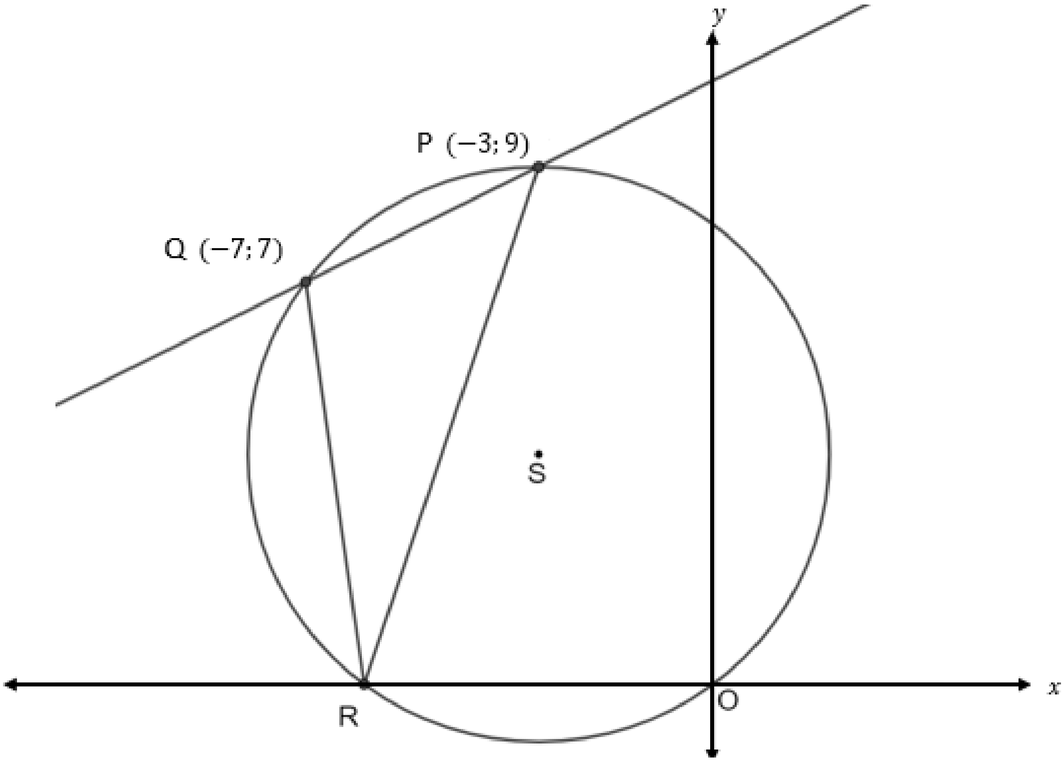
3.5 Calculate the coordinates of point Q, which lies in the second quadrant and forms a rectangle ACEQ in that order. (4)

[18]

QUESTION 4

In the diagram:

- Circle with centre S passes through points P (-3;9), Q (-7;7), R and the origin, O.
- R lies on the x-axis.
- The equation of the circle is: $x^2 + 6x + y^2 - 8y = 0$.



4.1 Determine the coordinates of centre S. (3)

4.2 Write down the radius of circle S. (1)

4.3 Calculate the coordinates of R. (2)

4.4 Determine the equation of the tangent to the circle at point Q. (4)

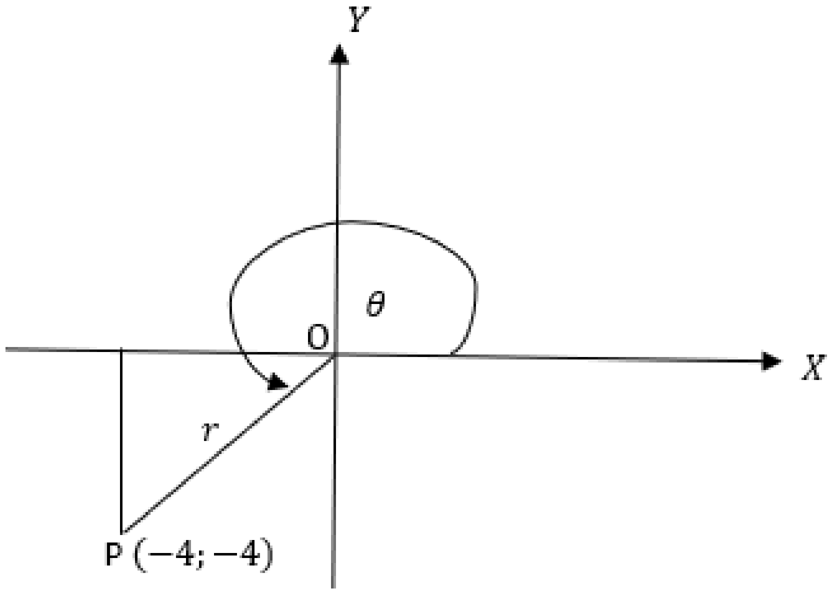
4.5 Determine whether line SQ passes through the midpoint, T, of PR. Show all your calculations. (5)

4.6 A second circle, with midpoint A and equation $(x-3)^2+(y+4)^2=25$, is drawn on the same set of axes. Calculate if the circles will touch externally or intersect one another. (4)

[19]

QUESTION 5

5.1 In the diagram $P(-4;-4)$ is given. $X\hat{O}P = \theta$.
Use the diagram to answer the questions that follow, **without the use of a calculator.**



5.1.1 Determine $\cos\theta$. (2)

5.1.2 Determine $\sin 2\theta - 1$. (3)

5.2 θ and β are both acute angles. If $\tan\theta = 4$ and $\tan\beta = 1$, calculate, **without the use of a calculator** the numerical value of:

$$\cos(\theta + \beta)$$

(5)

5.3 Simplify, **without the use of a calculator,** the following expression to a single trigonometric function:

$$\frac{\sin(90° + A)\sin 170°}{\sin 100° \cos(-A)\sin 90°}$$

(6)

5.4

5.4.1 Prove the following identity:

$$\frac{\cos\beta}{\cos\beta - \sin\beta} + \frac{\sin\beta}{\sin\beta + \cos\beta} = 1 + \tan 2\beta$$ (5)

5.4.2 Hence, determine, correct to one decimal place, the general solution for β, if:

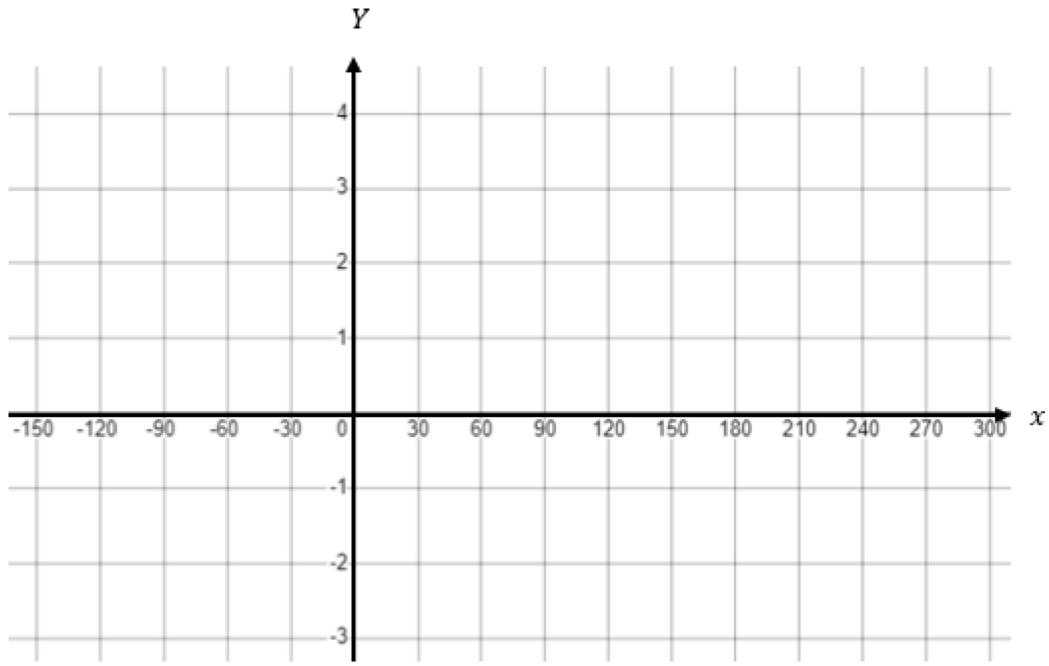
$$\frac{\cos\beta}{\cos\beta - \sin\beta} + \frac{\sin\beta}{\sin\beta + \cos\beta} = -4$$ (3)

[24]

QUESTION 6

6.1

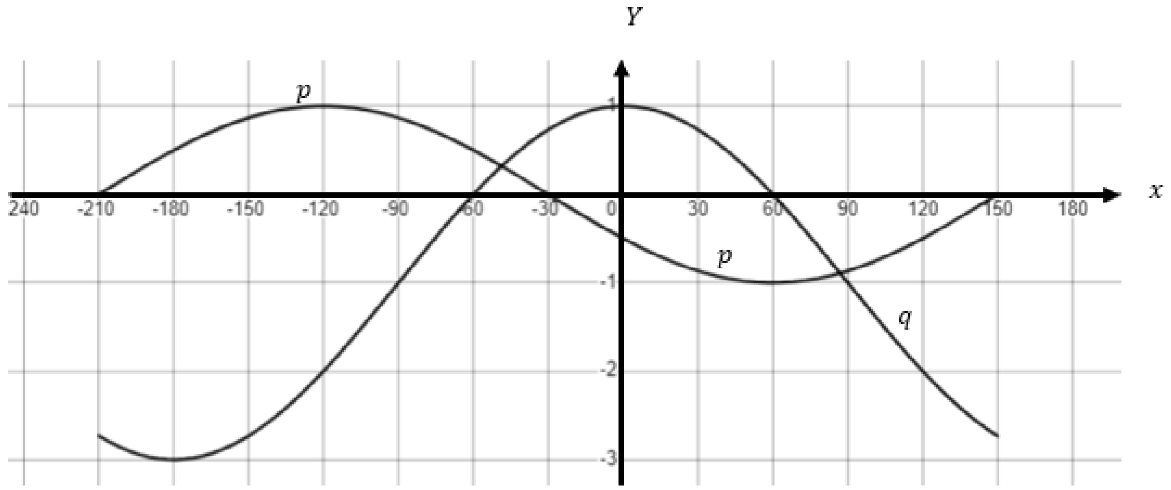
6.1.1 Sketch the graph of $f(x) = 3\sin x + 1$ for $x \epsilon [-90°; 270°]$ on the set of axes provided below. Clearly indicate the y-intercept and the turning point(s) on your graph. (3)



6.1.2 If $g(x) = 1$, use your graphs to determine the value(s) of x, in the interval $x\epsilon[-90°; 270°]$ where $g(x) = f(x)$. (2)

6.1.3 Write down the maximum value of $g(x) - f(x)$. (1)

6.2 The functions of $p(x) = a\sin(x+b)$ and $q(x) = 2\cos x + d$ for $x \epsilon [-210°\,; 150°]$ are given in the graph below.



6.2.1 Write down the amplitude of q. (1)

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6.2.2 Write down the value of a, b and d. (3)

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6.2.3 Use your graph to determine the values(s) of x if $q(x) - p(x) = 1$ for $x\epsilon\ [0°\,; 150°]$. (2)

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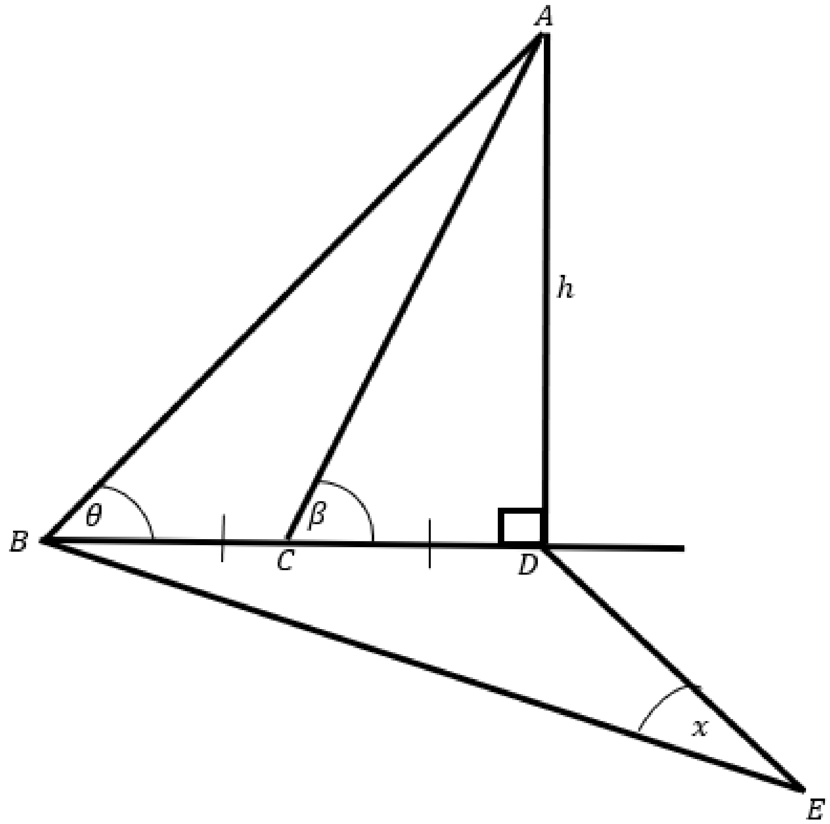
[12]

QUESTION 7

A cell tower, AD, with height h, is observed by two tourists standing at points B and C respectively. $BC = CD$. A third tourist is standing in the same horizontal plane as B and C, at point E. B and E are equidistant from the foot of the tower, $BD = DE$. $D\hat{E}B = x$. The angle of elevation from B to the top of the tower, A, is θ and the angle of elevation from C to A is β.



7.1 Write CD in terms of β and h. (2)

7.2 Prove that:

$$BE = \frac{h\sqrt{8(1 + cos2x)}}{\tan \beta}$$ (7)

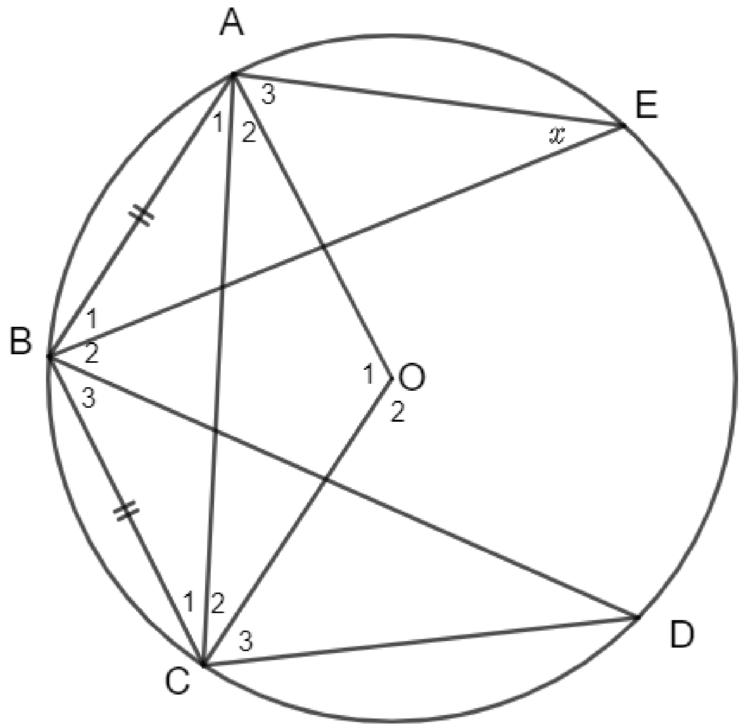
7.3 If $\beta = 56°$, determine the size of θ. (3)

[12]

QUESTION 8

In the diagram:

- O is the centre of the circle.
- The circle passes through A, B, C, D and E.
- AB = BC
- $\hat{E} = x$.



8.1 State, with reasons, THREE angles equal to x. (6)

8.2 If given that $\hat{O}_1 = 96°$, determine, with reasons, the size of $\hat{A}_2$. (3)

8.3 Determine, with reasons, the size of $A\hat{B}C$. (2)

8.4 Hence, or otherwise, determine the value of x. (2)

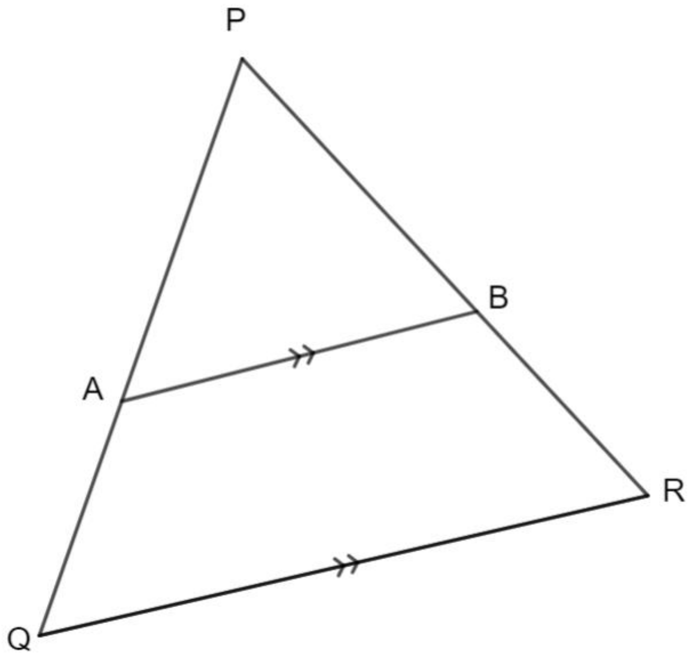
8.5 Determine, with reasons, if ABCO is a cyclic quadrilateral. (2)

[15]

QUESTION 9

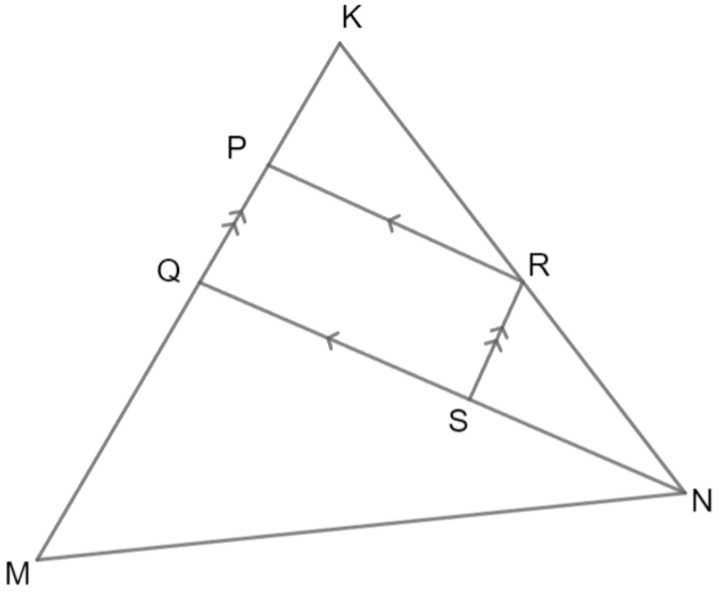
9.1 Use the diagram given below to prove the theorem that states that a straight line parallel to one side of a triangle divides the other sides proportionally.

Prove that: $\frac{PA}{AQ} = \frac{PB}{BR}$ (6)



9.2 In the diagram △KMN is drawn with P and Q points on KM and R a point on KN with:

- KQ = QM.
- PR ∥ QN.
- S is a point on QN with KM ∥ RS.



9.2.1 Prove that $\frac{NS}{SQ} = \frac{QP}{PK}$. (2)

9.2.2 Hence, determine QP : QK, if given $\frac{SQ}{NS} = \frac{1}{2}$. (2)

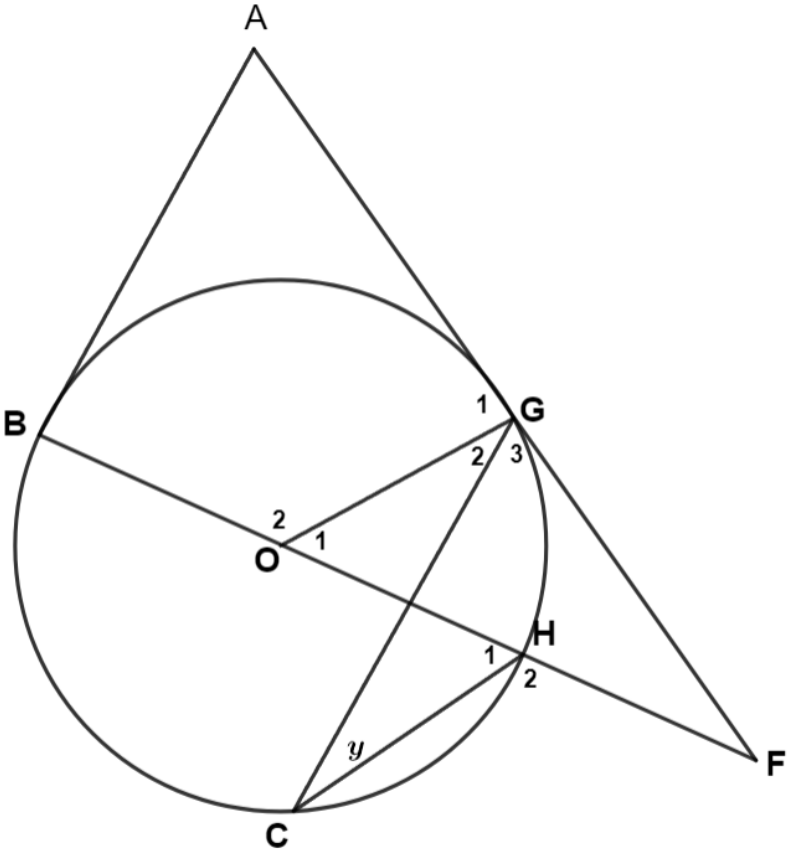
9.2.3 Determine the numerical value of RS : KM. (4)

[14]

QUESTION 10

In the diagram:

- AB and AG are tangents to the circle with centre O.
- Diameter BH produced to F.
- Tangent AG is produced to meet BOF at F.
- $G\hat{C}H = y$.



10.1 Prove that ABOG is a cyclic quadrilateral. (3)

10.2 Determine $\hat{A}$ in terms of y. (2)

10.3 Prove that ΔABF ||| ΔOGF. (4)

10.4 Prove that: $OF = \frac{BO \cdot AF}{AG}$ (5)

[14]

GRAND TOTAL: [150]

ADDITIONAL WRITING SPACE

END OF DOCUMENT

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$