MARKING GUIDELINES

EXAMINATION	NATIONAL SENIOR CERTIFICATE
GRADE	12
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SUBJECT	MATHEMATICS
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DURATION (HOURS)	3
NUMBER OF PAGES	27

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QUESTION 1

Pamela recorded the amount of data in MB that she used on each of the first 15 days in May. The information is shown in the table below:

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40

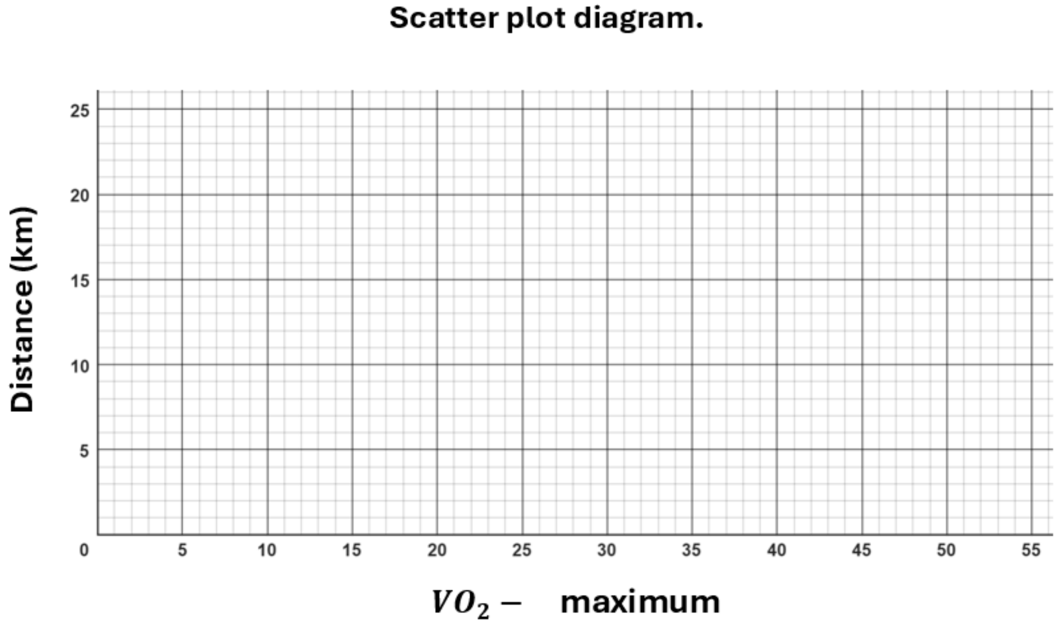
1.1	Calculate the mean for the data set.	(1)
	$\bar{x} = \frac{375}{15} = 25$	✓ Answer (1)
1.2	Determine the standard deviation for the data set.	(1)
	$\sigma = 17{,}65$	✓ Answer (1)
1.3	Determine the number of days for which the data used was greater than one standard deviation above the mean.	(2)
	$Upper\ value = \bar{x} + \sigma$ $Upper\ value = 25 + 17{,}65$ $Upper\ value = 42{,}65$ ∴ 2 values above one standard deviation.	✓ 42,65 ✓ 2 values (2)
		[4]

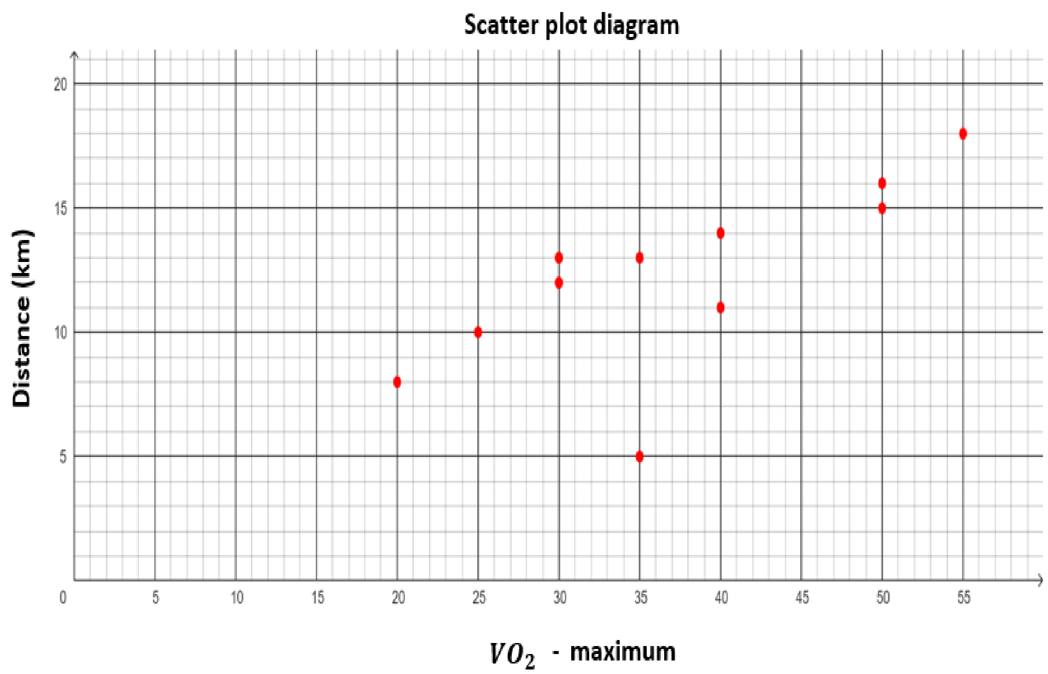
QUESTION 2

An athlete's ability to take in and use oxygen is known as their VO_2-maximum.
The table below shows eleven athletes' VO_2 maximum and the distance they ran in an hour.

VO_2-max	50	55	20	30	40	25	30	50	40	35	35
Distance (km)	15	18	8	13	14	10	12	16	11	13	5

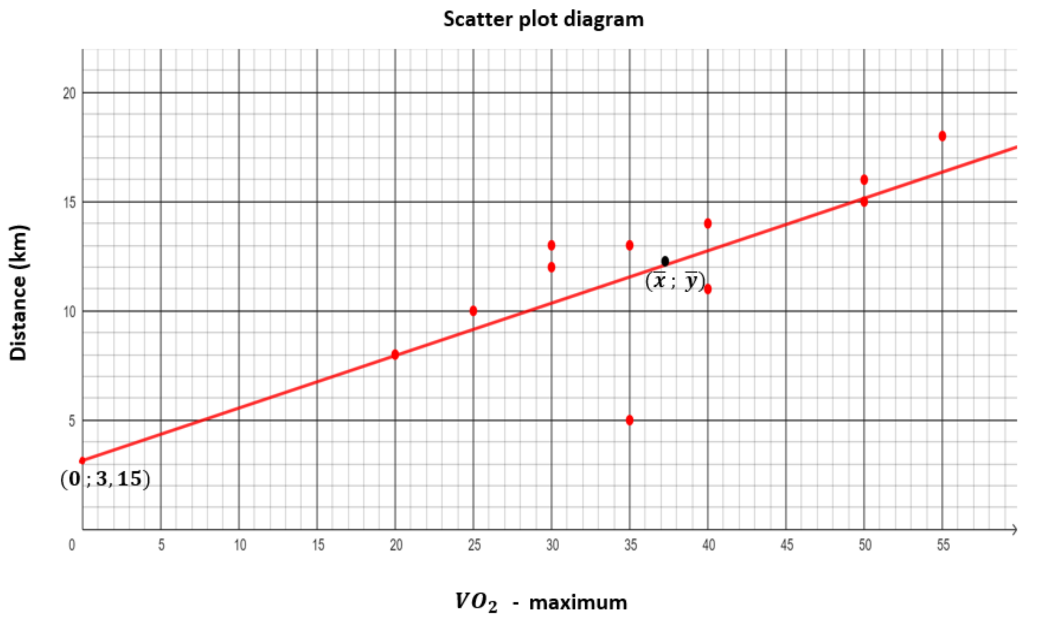
2.1	Draw a scatter plot of the above data on the below set of axes.	(2)





✓ (20;8) and (55;18)
✓ other points plotted correctly.

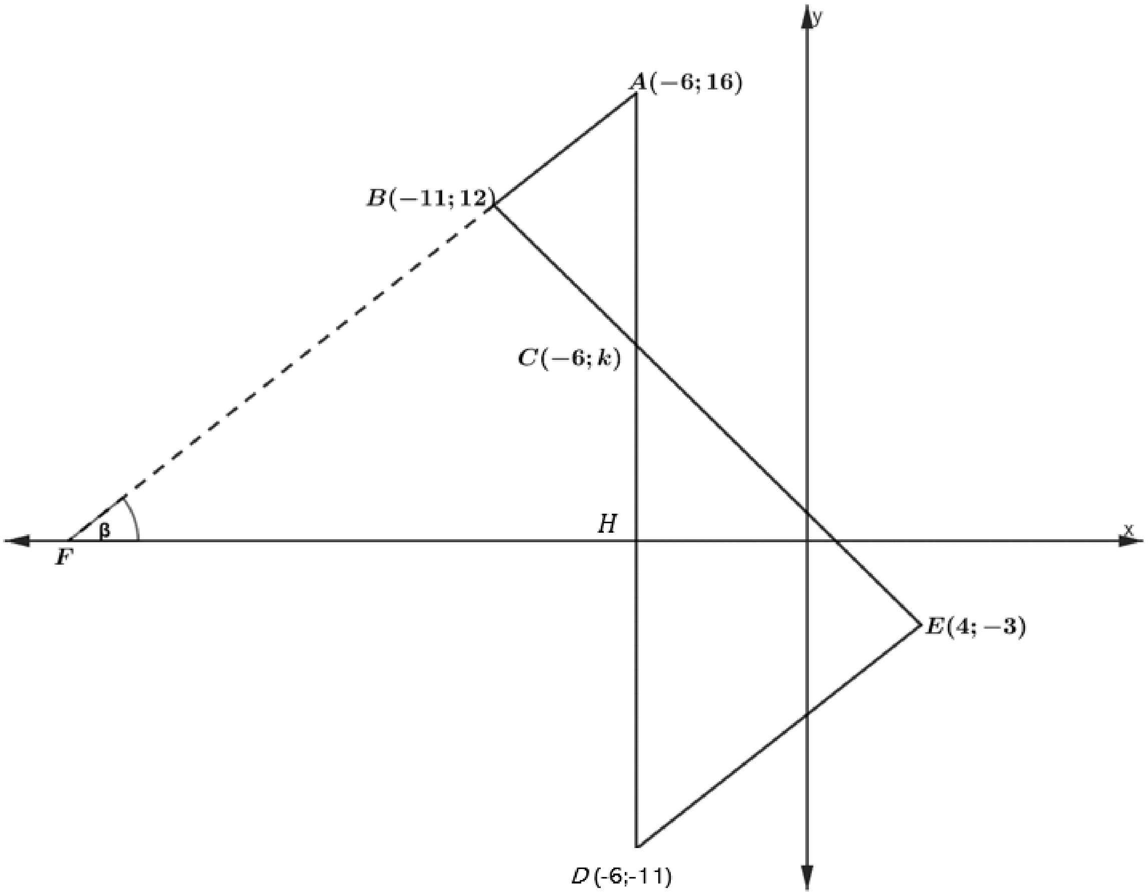
(2)

2.2	Calculate the equation of the least squares regression line of the data.	(3)
	$A = 3{,}15$ $B = 0{,}24$ $y = 0{,}24x + 3{,}15$	✓ $A = 3{,}15$ ✓ $B = 0{,}24$ ✓ Correct equation (3)
2.3	Draw the least square regression line on the scatter plot drawn in **QUESTION 2.1** above.	(3)
		✓ y-intercept ✓ ONE value $(\overline{37{,}27};\overline{12{,}27})$ OR (20;8) OR (55;16,4) ✓ Correct line (3)
2.4	Identify if there are any outliers in the data. Provide a reason for your answer.	(2)
	(35;5) is an outlier. This value deviates from all the other data in this set.	✓ (35;5) ✓ Reason
2.5	Predict the VO_2 maximum of an athlete who ran 19 km.	(2)
	$19 = 0{,}24x + 3.15$ $x = 66{,}04$	✓ Sub $y = 19$ ✓ $x = 66{,}04$ (2)
2.6	Determine the correlation coefficient of the data and comment on the correlation.	(2)
	$r = 0{,}73$ Moderate positive correlation	✓ r=0,73 ✓ moderate positive (2)
		[14]

QUESTION 3

In the diagram below:

- $A(-6;16)$; $B(-11;12)$; $D(-6;-11)$ and $E(4;-3)$ are given.
- AD intersects BE at point $C(-6;k)$.
- AB produced meets the x-axis in F.
- $H\hat{F}A = \beta$.
- AD intersects the x-axis at H.
- ACD and BCE are straight lines.



3.1	Show that the value of $k = 7$.	(4)
	$m_{BE} = \frac{12-(-3)}{-11-4}$ $m_{BE} = -1$ $\therefore m_{BC} = \frac{12-k}{-11-(-6)}$ $m_{BC} = -1$ $12 - k = 5$ $k = 7$	✓ Subst in corr. Formula ✓ $m_{BE} = -1$ ✓ Subst (-6;k) ✓ Simplify

	OR $m_{BE}=\frac{12-(-3)}{-11-4}$ $m_{BE}=-1$ $y=mx+c$ or $y-y_1=m(x-x_1)$ $-3=-(4)+c$ $(y-(-3))=-(x-4)$ $c=1$ $y+3=-x+4$ $\therefore y=-x+1$ $y=-x+1$ $C(-6;k)$: $k=-(-6)+1=7$ **OR** $DE=\sqrt{(4-(-6))^2+(-3-(-11))^2}$ $DE=\sqrt{164}$ $DE=2\sqrt{41}$ units $\frac{AC}{CD}=\frac{AB}{ED}$. . . line || one side of △ $\frac{16-k}{k-(-11)}=\frac{\sqrt{41}}{2\sqrt{41}}=\frac{1}{2}$ $2(16-k)=k+11$ $3k=21$ $k=7$	**OR** ✓ Subst in corr. Formula ✓ $m_{BE}=-1$ ✓ $y=-x+1$ ✓ Subst (-6;k) **OR** ✓ Subst in corr. Formula ✓ CE=$2\sqrt{41}$ ✓$\frac{AC}{CD}=\frac{AB}{ED}$ ✓ $\frac{16-k}{k-(-11)}=\frac{\sqrt{41}}{2\sqrt{41}}$
3.2	Calculate the length of BC.	(2)
	$BC^2=(12-7)^2+(-11+6)^2$ $BC=5\sqrt{2}$	✓ subst in corr formula ✓ Answer

<table>
<tr><td>3.3</td><td>Show that C is NOT the midpoint of BE.</td><td>(2)</td></tr>
<tr><td></td><td>$Midpoint\ BE = \left(\frac{-11+4}{2}\ ;\ \frac{12-3}{2}\right)$

$Midpoint\ BE = \left(-\frac{7}{2}\ ;\ \frac{9}{2}\right)$

C = (-6;7), ∴ NOT midpoint of BE</td><td>✓ Subst in correct formula

✓ $\left(-\frac{7}{2}\ ;\ \frac{9}{2}\right)$
(2)</td></tr>
<tr><td>3.4</td><td>Calculate the size of $B\hat{A}D$.</td><td>(5)</td></tr>
<tr><td></td><td>$m_{AB} = \frac{16-12}{-6+11} = \frac{4}{5}$

$\tan B\hat{F}X = m_{AB} = \frac{4}{5}$

$B\hat{F}X = tan^{-1}\left(\frac{4}{5}\right) = 38,6598 \ldots \approx 38,66°$

$\beta = 180° - 38,66° = 141,34°$

$B\hat{A}D = 141,34 - 90°$ . . . . . ext $\angle$ $of\ \Delta = sum\ opp\ int\ \angle$'s

$B\hat{A}D = 51,34°$

OR

$AD \perp x-axis$ m_{AD} is undefined.

$\therefore F\hat{H}A = 90^o$

$m_{AB} = \frac{4}{5}$ from 3.2

$\therefore A\hat{F}H = 38,66^o$

$\therefore B\hat{A}D = 180^o - 90^o - 38,66^o$
$= 51,34^o$</td><td>✓ m_{AB}
✓ $\tan B\hat{F}X = \frac{4}{5}$

✓ $B\hat{F}X = 38,66°$

✓ $\alpha = 141,34°$
✓ $B\hat{A}D = 51,34°$

OR

✓ $F\hat{H}A = 90^o$

✓ $m_{AB} = \frac{4}{5}$

✓ $A\hat{F}H = 38,66^o$

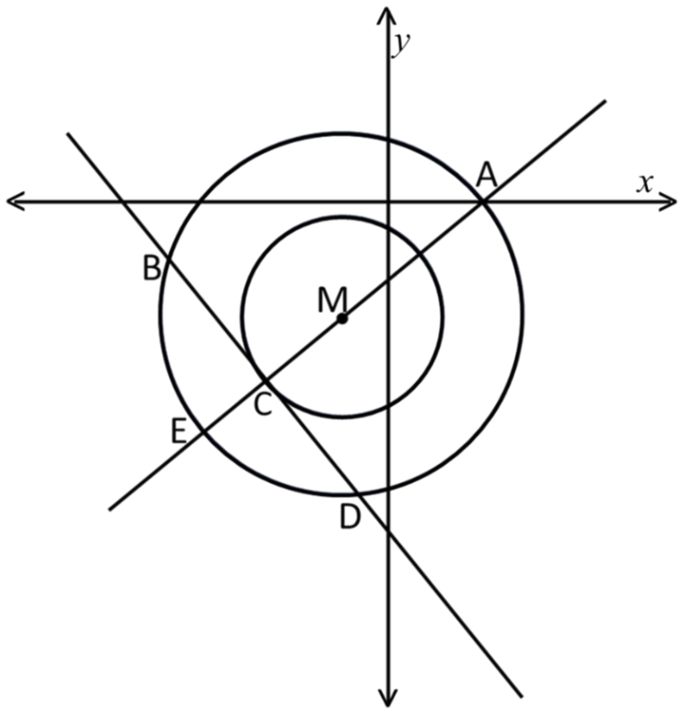
✓ $180^o - 90^o - 38,66^o$
✓ $B\hat{A}D = 51,34°$</td></tr>
</table>

3.5	Calculate the numerical value of $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEC}$	(5)
	$AC = 16 - 7 = 9$ units (k-value from 3.1) $DC = 7 + 11 = 18$ units $BC = 5\sqrt{2}$ $EC = \sqrt{(-6-4)^2 + (7+3)^2} = 10\sqrt{2}$ $B\hat{C}A = E\hat{C}D$ . . . vertically opposite angles $\frac{Area\ \Delta ABC}{Area\ \Delta DEC} = \frac{\cancel{\frac{1}{2}} \times BC \times AC \times \cancel{\sin B\hat{C}A}}{\cancel{\frac{1}{2}} \times DC \times CE \times \cancel{\sin D\hat{C}E}}$ $\frac{Area\ \Delta ABC}{Area\ \Delta DEC} = \frac{(5\sqrt{2})(9)}{(18)(10\sqrt{2})}$ $\frac{Area\ \Delta ABC}{Area\ \Delta DEC} = \frac{1}{4}$	✓ DC = 18 ✓ EC=$10\sqrt{2}$ ✓ $B\hat{C}A = E\hat{C}D$ ✓ area-rule subst. ✓ Answer
		[18]

QUESTION 4

In the diagram below:

- The small circle (P_1) and the big circle (P_2) have the same center M.
- A and E are points on circle P_2.
- EM intersects circle P_1 at point C.
- The tangent BD to circle P_1 intersects circle P_2 at B and D.
- The equation of circle P_1 is given by $x^2 + 2x + y^2 + 10y + 6 = 0$.
- The equation of line EM is $y = x - 4$.

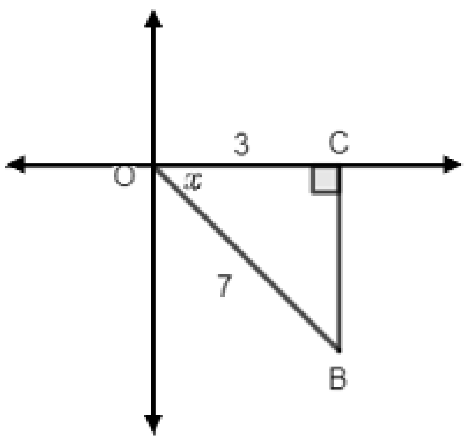


4.1	Give the coordinates of centre M.	(3)
	$(x+1)^2 + (y+5)^2 = -6 + 1 + 25$ $(x+1)^2 + (y+5)^2 = 20$ $M\ (-1; -5)$	✓ $(x+1)^2$ ✓ $(y+5)^2$ ✓ (-1;-5)
4.2	Determine the radius of circle P_1.	(1)
	$r = \sqrt{20}$ (accept $2\sqrt{5}$ or 4,47)	✓ Answer

4.3	C is the point where line EM and circle P_1 intersect. Determine the coordinates of C, correct to two decimal places. Show all workings.	(6)
	Given: $y = x - 4$ $(x+1)^2 + (x-4+5)^2 = 20$ $x^2 + 2x + 1 + x^2 + 2x + 1 = 20$ $2x^2 + 4x + 2 - 20 = 0$ $2x^2 + 4x - 18 = 0$ $x^2 + 2x - 9 = 0$ $x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-9)}}{2(1)}$ $x = \frac{-2 \pm \sqrt{40}}{2}$ $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$ $x = 2{,}16$ N/A $x = -4{,}16$ $y = (-4{,}16) - 4$ $y = -8{,}16$ C(-4,16 ; -8,16)	✓ Substitution ✓ $2x^2 + 4x - 18 = 0$ subst in quadratic eq ✓ $x = -4{,}16$ ✓ N/A $x = 2{,}16$ ✓ C(-4,16 ; -8,16) (y –value)
4.4	Determine the gradient of the tangent to circle P_2 at point E.	(3)
	$M\hat{C}D = 90°$ . . . radius ⊥ tangent $m_{ME} = 1$ (given) $\therefore m_{tangent} = -1$ . . . radius ⊥ tangent **OR** $m_{ME} = \frac{-8{,}16 + 5}{-4{,}16 + 1}$ $m_{ME} = 1$ $m_{BC} \times m_{ME} = -1$ . . . radius ⊥ tangent $\therefore m_{tangent} = -1$	✓ $M\hat{C}D = 90°$ ✓ $m_{ME} = 1$ ✓ Answer **OR** ✓ $m_{ME} = 1$ ✓ $m_{BC} \times m_{ME} = -1$ ✓ Answer
4.5	If it is given that $CD = 5\sqrt{2}$, determine MD, the radius of circle P_2.	(4)
	$MD^2 = CD^2 + MC^2$ . . . Pythagoras $MD^2 = (5\sqrt{2})^2 + (\sqrt{20})^2$ $MD = \sqrt{70}$ units	✓ S ✓ $(5\sqrt{2})^2$ ✓ $(\sqrt{20})^2$ ✓ $MD = \sqrt{70}$ Accept 8,37 units

4.6	Write down the equation of P2 in the form $(x-a)^2+(y-b)^2=r^2$.	(1)
	$(x+1)^2+(y+5)^2=70$	✓ Answer
4.7	Prove that point $q(\sqrt{5}\,;0)$ lies inside circle P2. Show all your calculations.	(4)
	Distance from q to centre: $=\sqrt{(\sqrt{5}+1)^2+(0+5)^2}$ $=5{,}96$ $\sqrt{70}=8{,}37$ $5{,}96<8{,}37\ (\sqrt{70})$ $\therefore q(\sqrt{5};0)$ lies inside the circle C2.	✓ correct subst ✓ 5,96 ✓ 8,37 ✓ $5{,}96<8{,}37\ (\sqrt{70})$
		[22]

QUESTION 5

5.1	If $7\cos x = 3$ and $x \epsilon [90°\,; 360°]$, calculate the value of $3\cos 2x$ **without the use of a calculator**.	(4)
	$\cos x = \frac{3}{7}$ $3\cos 2x = 3(2\cos^2 x - 1)$ $= 3\left[2\left(\frac{3}{7}\right)^2 - 1\right]$ $= -\frac{93}{49}$	✓ $\cos x = \frac{3}{7}$ ✓ Identity ✓ Sub corr ratio ✓ Answer
	OR	**OR**
	$\cos x = \frac{3}{7}$ $y = \sqrt{(7)^2 - (3)^2}$ $y = 2\sqrt{10}$ $3\cos 2x = 3(1 - 2\sin^2 x)$ $= 3\left[1 - 2\left(\frac{2\sqrt{10}}{7}\right)^2\right]$ $= -\frac{93}{49}$ 	✓ $\cos x = \frac{3}{7}$ ✓ Identity ✓ Sub corr ratio ✓ Answer
	OR	**OR**
	$\cos x = \frac{3}{7}$ $3\cos 2x = 3(\cos^2 x - \sin^2 x)$ $= 3\left[\left(\frac{3}{7}\right)^2 - \left(\frac{2\sqrt{10}}{7}\right)^2\right]$ $= -\frac{93}{49}$	✓ $\cos x = \frac{3}{7}$ ✓ Identity ✓ Sub corr ratio ✓ Answer (4)

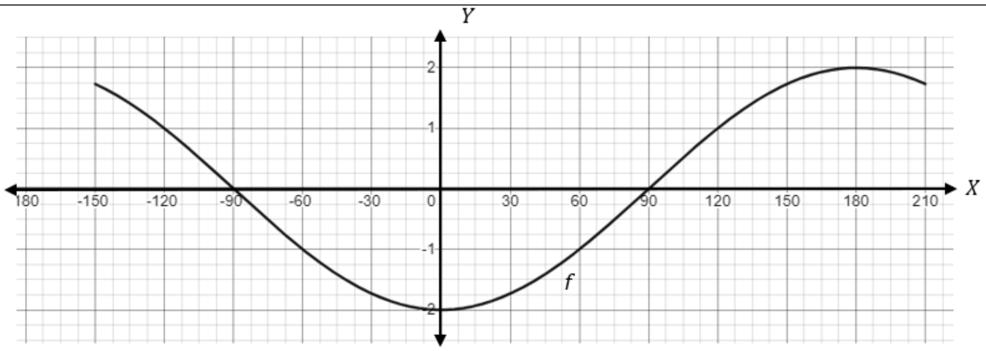
5.2	Prove that $2\sin x - \cos^2 x$ can be written as $(\sin x + 1)^2 - 2$.	(3)
	$(\sin x + 1)^2 - 2$ $= \sin^2 x + 2\sin x + 1 - 2$ $= 2\sin x - 1 + \sin^2 x$ $= 2\sin x - (1 - \sin^2 x)$ $= 2\sin x - \cos^2 x$ **OR** $2\sin x - \cos^2 x$ $= 2\sin x - (1 - \sin^2 x)$ $= 2\sin x - 1 + \sin^2 x$ $= \sin^2 x + 2\sin x - 1$ $= \sin^2 x + 2\sin x + 1 - 2$ $= (\sin x + 1)^2 - 2$	✓ quadratic trinomial ✓ Simplify ✓ Identity **OR** ✓ Identity ✓ quadratic trinomial ✓$(+1)\ (-2)$

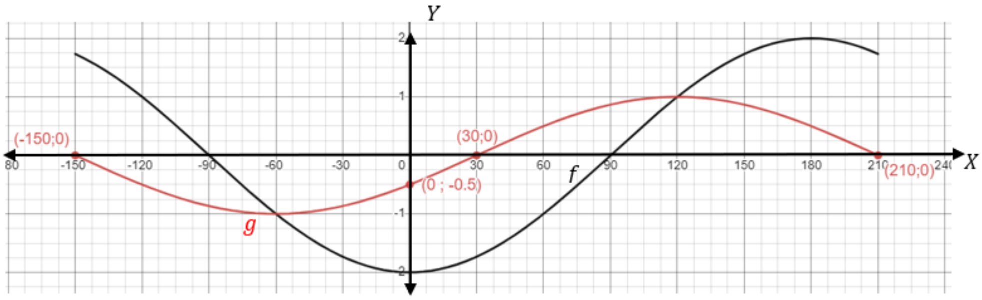
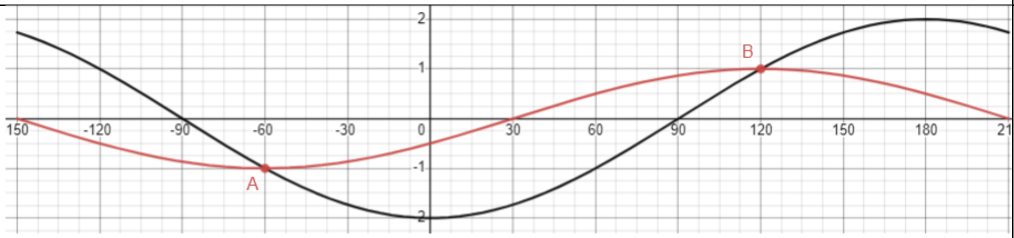
5.3.1	Determine the general solution for the value(s) of x if: $5\sin 2x - \cos 2x = 0$	(5)
	$\frac{5\sin 2x}{\cos 2x} - \frac{\cos 2x}{\cos 2x} = 0$ $5\tan 2x - 1 = 0$ $\tan 2x = \frac{1}{5}$ RA= 11,31° $2x = 11{,}31° + k \cdot 180°\ ; k\epsilon Z$ $x = 5{,}66° + k \cdot 90°; k\epsilon Z$ $\therefore x = 5{,}66° + k \cdot 90°; k\epsilon Z$ (Accept 5,65)	✓ $\div \cos 2x$ ✓ $\tan 2x = \frac{1}{5}$ ✓ RA=11,31° ✓ 1 mark for $x = 5{,}66° + k \cdot 90°$ ✓1 mark for $k \in Z$

5.3.2	For which value(s) of x will $5\tan 2x - 1 = 0$ and $x\epsilon[-180°; 180°]$ be undefined?	(2)
	$x\epsilon\{-135°\,;\,-45°\,;45°\,;135\}$	✓ 1 mark TWO values ✓ 1 mark OTHER TWO values
5.4	Simplify to a single trigonometric function, **without the use of a calculator**: $$-1 + \cos(180° - \theta).\sin(\theta - 90°)$$	(3)
	$-1 + \cos(180° - \theta).\sin(\theta - 90°)$ $= -1 + (-\cos\theta).(-\cos\theta)$ $= -1 + \cos^2\theta$ $= -(1 - \cos^2\theta)$ $= -\sin^2\theta$	✓ - cos θ ✓ - cos θ ✓ Answer
5.5	If $\cos 44° = \sqrt{p}$, determine $\sin^2 68°$ in terms of p, **without the use of a calculator.**	(4)
	$\cos(2(22°)) = \sqrt{p}$ $2\cos^2 22° - 1 = \sqrt{p}$ $2\cos^2\ (90° - 68°) - 1 = \sqrt{p}$ $2\sin^2 68° - 1 = \sqrt{p}$ $\sin^2 68° = \frac{\sqrt{p}+1}{2}$ **OR** $\cos 136° = 1 - 2\sin^2 68$ $2\sin^2 68° = 1 - \cos 136°$ $= 1 + \cos 44°$ $\sin^2 68° = \frac{\sqrt{p}+1}{2}$	✓ cos (2(22)) ✓ Identity ✓ Reduction (90-68) ✓ Answer **OR** ✓ $1 - 2\sin^2 68$ ✓ Simplify ✓ Acute angle ✓ Answer

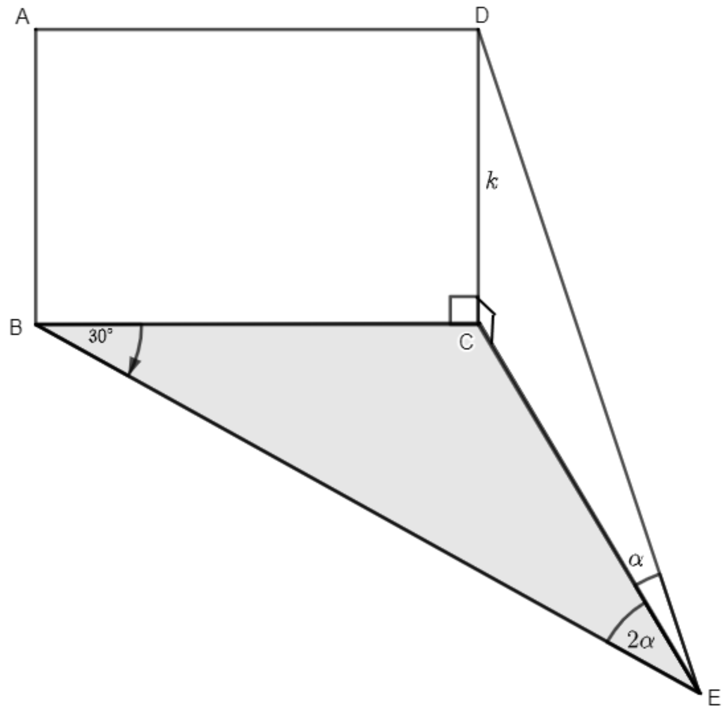
5.6	Prove the following identity:	
	$$\frac{\sin 7x \cdot \cos 5x - \cos 7x \cdot \sin 5x}{\tan 2x} - 1 = -2\sin^2 x$$	(4)
	$LS = \frac{\sin(7x - 5x)}{\tan 2x} - 1$ $LS = \frac{\sin 2x}{\frac{\sin 2x}{\cos 2x}} - 1$ $LS = \sin 2x \times \frac{\cos 2x}{\sin 2x} - 1$ $LS = \cos 2x - 1$ $LS = 1 - 2\sin^2 x - 1$ $LS = -2\sin^2 x = RS$	✓ $\sin(7x - 5x)$ ✓ $\frac{\sin 2x}{\cos 2x}$ ✓ $\cos 2x - 1$ ✓ 1-2$\sin^2 x$
5.7	Given: $\tan x = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots}}}}$ Prove that: $\sin x = \sqrt{\cos x(\cos^2 x + \sin x)}$	(5)
	$(\tan x)^2 = \left(\sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots}}}}\right)^2$ $\tan^2 x = \cos x + \tan x$ $\frac{\sin^2 x}{\cos^2 x} = \cos x + \frac{\sin x}{\cos x}$ $\sin^2 x = \cos^3 x + \sin x \cos x$ $\sin^2 x = \cos x(\cos^2 x + \sin x)$ $\sin x = \sqrt{\cos x(\cos^2 x + \sin x)}$	✓ square both sides ✓ Subst $\tan x$ ✓ tan identity ✓ LCD ✓ $\sin^2 x = \ldots$
		[30]

QUESTION 6

6.1	Determine the value(s) of x **without the use of a calculator**, if $\sin(x-30°)+2\cos x=0; x\epsilon[-150°;210°]$ and $\cos x \neq 0$.	(6)
	$\sin x\cdot\cos 30° - \cos x\cdot\sin 30° + 2\cos x = 0$ $\sin x\cdot\left(\frac{\sqrt{3}}{2}\right) - \cos x\cdot\left(\frac{1}{2}\right) + 2\cos x = 0$ $\frac{\sqrt{3}}{2}\sin x + \frac{3}{2}\cos x = 0$ $\frac{\sqrt{3}}{2}\frac{\sin x}{\cos x} + \frac{3\cos x}{2\cos x} = \frac{0}{\cos x}$ $\frac{\sqrt{3}}{2}\tan x = -\frac{3}{2}$ $\tan x = -\frac{3}{\sqrt{3}}$ $RA = 60°$ $x = 180° - 60° + k.180°; k\epsilon Z$ OR $x = -60° + k.180°; k\epsilon Z$ $x\epsilon\{-60°\,;120°\}$	✓ compound id. ✓ $\frac{\sqrt{3}}{2}$ ✓ $\frac{1}{2}$ ✓ $\div \cos x$ ✓ $\tan x = -\frac{3}{\sqrt{3}}$ ✓ -60° AND 120°
6.2	The diagram below shows the graph of $f(x) = -2\cos x$, where $x\epsilon[-150°;210°]$.	
		

6.2.1	Sketch the graph of $g(x) = \sin(x - 30°)$ on the same diagram given above. Clearly show all intercepts with the axes as well as the turning points.	(4)
		✓ shape ✓y- intercept ✓ All x-intercepts ✓T/P (-60;-1) and (120;1)
6.2.2	Write down the amplitude of f .	(1)
	Amplitude = 2	✓ Answer
6.3	Use your graphs to determine the value(s) of x, in the interval $x\epsilon[-150°; 210°]$ such that:	
6.3.1	$g(x) - f(x) = 0$.	(2)
	At -60^o, $g(x) - f(x) = -1 - (-1) = 0$ At 120^o, $g(x) - f(x) = 1 - 1 = 0$ $x\epsilon\{-60°\,;120°\}$ OR $x = -60°$ or $x = 120°$	✓ $-60°$ ✓ $120°$
6.3.2	Indicate, using the symbols A and B, your answer to question 6.3.1 on the graph drawn in question 6.2.1	(1)
		✓ Answer
		[14]

QUESTION 7

<table>
<tr><td colspan="3">In the diagram B, C and E lie in the same horizontal plane. ABCD is a rectangular cardboard piece and CDE is a triangular cardboard piece with a right angle at C, and $DC = k$. The cardboard pieces are placed perpendicular to the horizontal plane as shown in the diagram.
The angle of elevation from E to D is α. $C\hat{E}B = 2\alpha$ and $E\hat{B}C = 30°$.</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>7.1</td><td>Write CE in terms of k and α.</td><td>(2)</td></tr>
<tr><td></td><td>$\frac{k}{CE} = \tan\alpha$
$CE = \frac{k}{\tan\alpha}$
OR
$\frac{CE}{\sin(90° - \alpha)} = \frac{k}{\sin\alpha}$
$CE = \frac{k\cos\alpha}{\sin\alpha}$</td><td>✓ tan ratio
✓ Answer
OR
✓ sin rule
✓ Answer</td></tr>
</table>

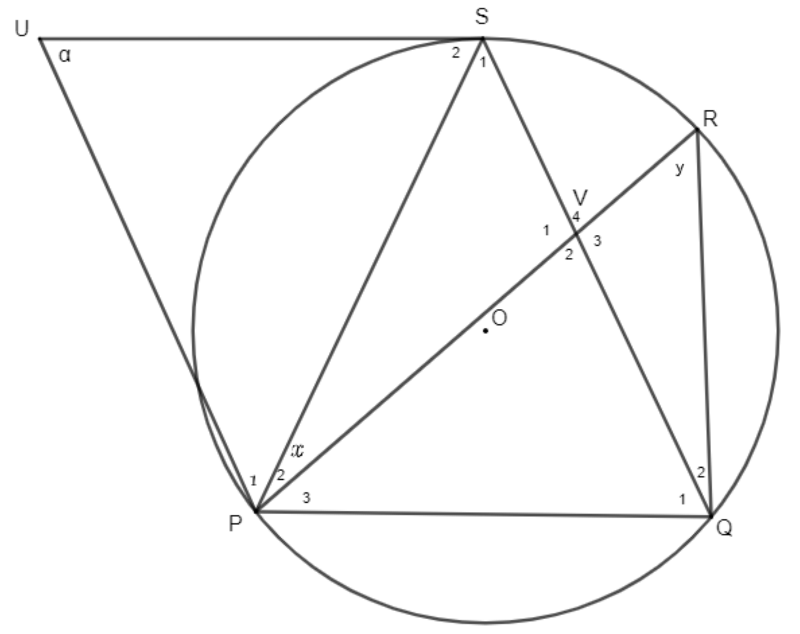
7.2	Prove that $BC = 4\,k\,\cos^2\alpha$	(4)
	$\frac{BC}{\sin 2\alpha} = \frac{CE}{\sin 30°}$ $BC = \frac{\frac{k}{\tan\alpha}\cdot 2\sin\alpha\cos\alpha}{\frac{1}{2}}$ $BC = \frac{4k\cdot\sin\alpha\cos\alpha}{\frac{\sin\alpha}{\cos\alpha}}$ $BC = 4k\cos^2\alpha$	✓ Sin rule ✓ double angle identity ✓ $\frac{1}{2}$ ✓ $\frac{\sin\alpha}{\cos\alpha}$

7.3	Show that the area of △BCE is equal to: $$\frac{\mathbf{2k^2\cos^3\alpha\,.\sin(30°+2\alpha)}}{\mathbf{\sin\alpha}}$$	(3)
	$Area\ \Delta BCE = \frac{1}{2}\cdot BC\cdot EC\cdot\sin(180° - (30° + 2\alpha))$ $Area\ \Delta BCE = \frac{1}{2}\cdot(4k\cos^2\alpha)\left(\frac{k}{\tan\alpha}\right)(\sin(30° + 2\alpha))$ $Area\ \Delta BCE = \frac{2k^2\cos^2\alpha\,.\sin(30° + 2\alpha)}{\tan\alpha}$ $Area\ \Delta BCE = \frac{2k^2\cos^2\alpha\,.\sin(30° + 2\alpha)}{\frac{\sin\alpha}{\cos\alpha}}$ $Area\ \Delta BCE = \frac{2k^2\cos^2\alpha\,.\sin(30° + 2\alpha)}{1}\times\frac{\cos\alpha}{\sin\alpha}$ $Area\ \Delta BCE = \frac{2k^2\cos^3\alpha\,.\sin(30°+2\alpha)}{\sin\alpha}$	✓ area rule ✓ Substitution ✓ $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$
		[9]

QUESTION 8

In the diagram:

- O is the centre of the circle.
- The circle passes through P, Q, R, and S.
- SQ and PR intersect at V.
- SU is a tangent to the circle at S.
- UP is drawn.
- $\hat{U} = \alpha$.
- $\hat{R} = y$.
- $\hat{P}_2 = x$.
- $\alpha = x + y$.



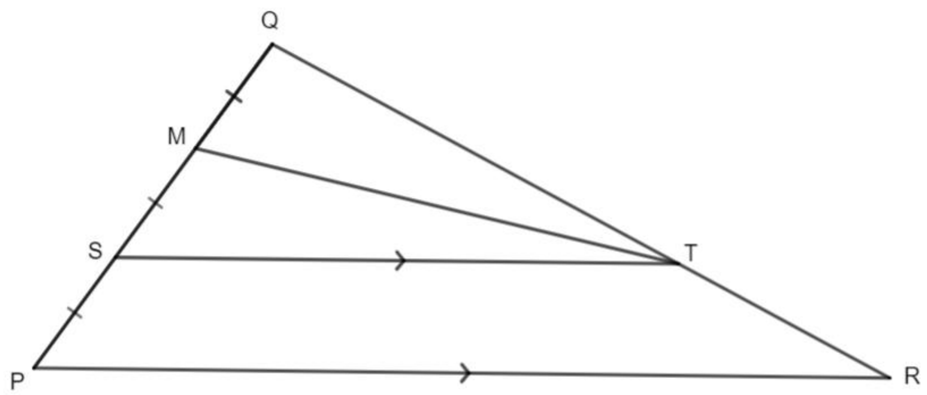
8.1	Give, with a reason, the size of:	(2)
8.1.1	$\hat{Q}_2$	
	$\hat{Q}_2 = x$ . . . ∠'s in same circle segment	✓ S ✓ R

8.1.2	$\hat{S}_1$	(1)
	$\hat{S}_1 = y$ . . . ∠'s in same circle segment	✓ S & R
8.2	Prove that VSUP is a cyclic quadrilateral.	(3)
	$\hat{V}_2 = x + y$ . . . Ext ∠ △SPV = sum opp int ∠'s $\hat{U} = \alpha = x + y$ . . . given ∴ $\hat{V}_2 = \hat{U}$ ∴ VSUP is cyclic quadrilateral . . . Converse ext ∠ = opp. Int ∠ **OR** $\hat{V}_2 = x + y$ . . . Ext ∠ △SPV = sum opp int ∠'s $\hat{V}_1 = 180° - (x + y)$ . . . ∠'s on straight line $\hat{V}_1 + \hat{V}_2 = 180° - (x + y) + (x + y)$ $\hat{V}_1 + \hat{V}_2 = 180°$ ∴ VSUP is cyclic quadrilateral . . . Converse Opp ∠'s supplementary	✓ S & R ✓ S ✓ R **OR** ✓ S & R ✓ S ✓ R
8.3	If $PQ = QR$, prove that PQ is a tangent to the circle through V, S, U and P.	(3)
	$\hat{P}_3 = y$ . . . ∠'s opposite = sides ∴ $\hat{S}_1 = \hat{P}_3 = y$ PQ is a tangent . . . converse tan-chord theorem	✓ S & R ✓ S ✓ R
		[9]

QUESTION 9

In the diagram △PQR is drawn with S and T points on PQ and QR respectively.

- M is a point on QS.
- ST ∥ PR
- PS = SM = MQ



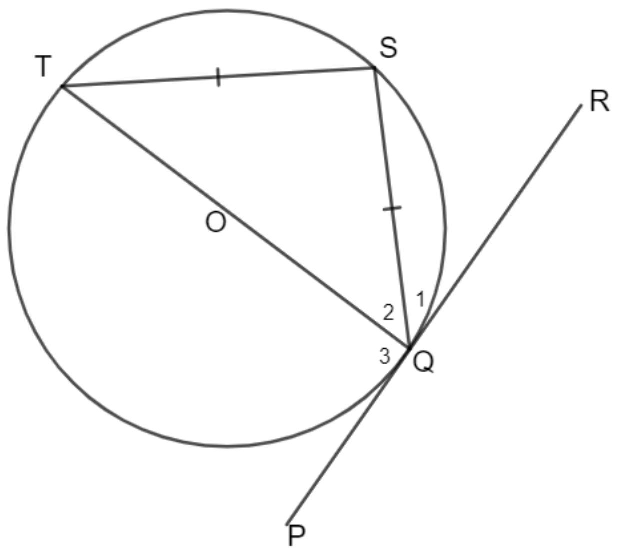
9.1	Let $SP = k$. Calculate $\frac{TR}{QT}$.	(2)
	$\frac{TR}{QT} = \frac{SP}{QS}$ . . . . line parallel to one side of △ $\frac{TR}{QT} = \frac{k}{2k} = \frac{1}{2}$	✓ S&R ✓ Answer
9.2	Determine the numerical value of $\frac{Area\ of\ \Delta QMT}{Area\ of\ \Delta QPR}$.	(4)
	$\frac{Area\ of\ \Delta QMT}{Area\ of\ \Delta QPR} = \frac{\frac{1}{2}\ QM \cdot QT \sin Q}{\frac{1}{2}\ QP \cdot QR \sin Q}$ $\frac{Area\ of\ \Delta QMT}{Area\ of\ \Delta QPR} = \frac{\frac{1}{2}\ (k)(2m) \sin Q}{\frac{1}{2}\ (3k)(3m) \sin Q}$ $\frac{Area\ of\ \Delta QMT}{Area\ of\ \Delta QPR} = \frac{2}{9}$	✓ area rule ✓½(k)(2m) sinQ ✓½(3k)(3m) sinQ ✓ Answer

9.3	If ST = 22 cm, determine the length of PR.	(4)
	In △PQR and △SQT: $\hat{Q} = \hat{Q}$ . . . common angle $Q\hat{P}R = Q\hat{S}T$ . . . corresponding ∠'s ; ST∥PR $Q\hat{R}P = Q\hat{T}S$ . . . corresponding ∠'s ; ST∥PR **OR** Sum int.∠'s of △ $\triangle PQR$ ||| $\triangle SQT$ . . . A;A;A $\therefore \frac{PR}{ST} = \frac{PQ}{SQ}$ . . . $\triangle PQR$ ||| $\triangle SQT$ $\frac{PR}{22} = \frac{3k}{2k}$ $PR = 33\ cm$	✓ common ✓ corresp. ∠ST∥PR ✓ S&R ||| ✓ Answer
		[10]

QUESTION 10

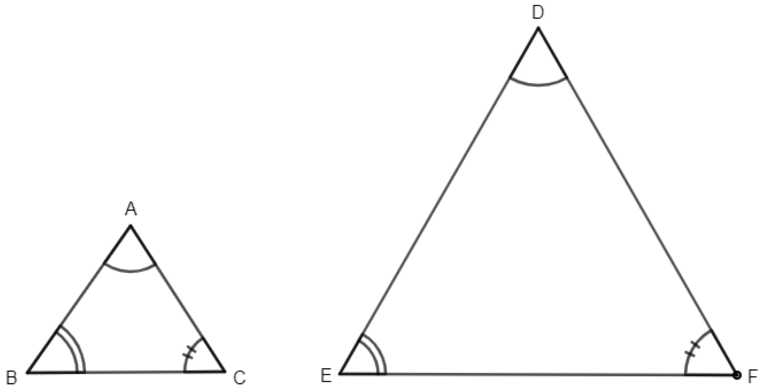
In the diagram:

- TOQ is a diameter of the circle with centre O.
- T, S and Q lie on the circumference of the circle.
- PR is a tangent to the circle at Q.
- TS = SQ.



Determine, with reasons, the size of $\hat{Q}_1$.	(4)
$\hat{Q}_2 = \hat{T}$ . . . ∠'s opposite = sides $\hat{T} = \hat{Q}_1$ . . . tan-chord $\therefore\ \hat{Q}_1 = \hat{Q}_2$ $\hat{Q}_1 + \hat{Q}_2 = 90°$ . . . radius ⊥ tangent $Q_1 = 45°$ **OR** $\hat{S} = 90°$ . . . ∠ in semi-circle $\hat{Q}_2 = \hat{T}$ . . . ∠'s opposite = sides $\hat{Q}_2 = 45°$ $\hat{T} = \hat{Q}_1$ . . . tan-chord $\therefore\ \hat{Q}_1 = 45°$	✓ S & R ✓ S & R ✓ S & R ✓ Answer **OR** ✓ S & R ✓ S & R ✓ S & R ✓ Answer
	[4]

QUESTION 11

<table>
<tr><td>11.1</td><td>In the diagram ΔABC and ΔDEF is given with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.
Use this diagram to prove that $\frac{AB}{ED} = \frac{BC}{EF}$.</td><td>(6)</td></tr>
<tr><td></td><td>
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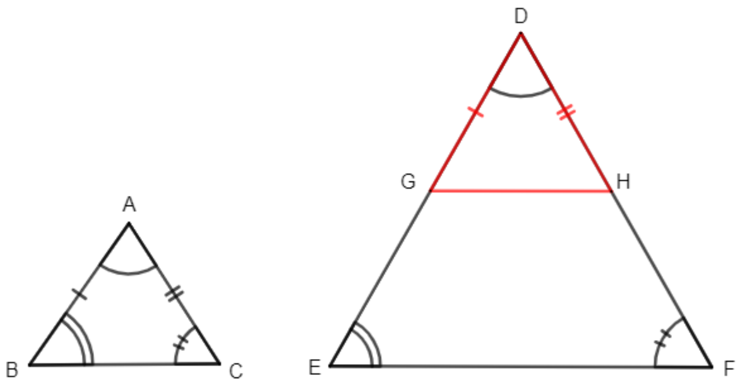
<tr><td></td><td>
</td><td></td></tr>
<tr><td></td><td>Construction: Draw GH such that AB=DG and AC=DH.
In ΔABC and △DGH:
AB = DG Construction
AC = DH Construction
$\hat{A} = \hat{D}$ Given
∴ ΔABC ≡ △DGH S; A; S

In △DEF:
$D\hat{G}H = \hat{B}$ ΔABC ≡ △DGH
But $\hat{B} = \hat{E}$ Given
$D\hat{G}H = \hat{E}$

∴ $GH \parallel EF$ Corresponding angles are equal

$\frac{DG}{DE} = \frac{DH}{DF}$ Line parallel to one side of △

But DG = AB
And DH = AC Construction
∴ $\frac{AB}{DE} = \frac{AC}{DF}$</td><td>✓ Construction

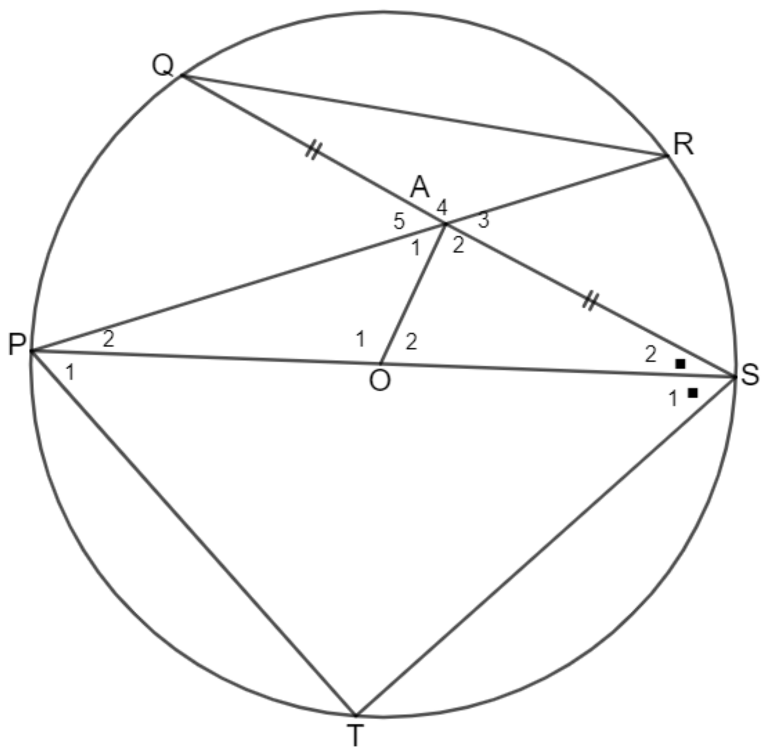
✓ ΔABC ≡ △DGH
✓S;A;S

✓ $D\hat{G}H = \hat{E}$

✓ $GH \parallel EF$ &

R

✓ R</td></tr>
</table>

11.2	In the diagram: • POS is the diameter of the circle with centre O. • QAS and PAR are straight lines. • P, Q, R, S and T lie on the circumference of the circle. • $QA = AS$. • $\hat{S}_1 = \hat{S}_2$. • SA produced meets the circle in Q. • QR, PT and ST are drawn.	
		
11.2.1	Write down, with a reason, the size of $\hat{T}$.	(2)
	$\hat{T} = 90°$ . . . ∠ in semicircle.	✓ 90° ✓ R

11.2.2	Prove that △PTS ||| △OAS.	(4)
	$\hat{A}_2 = 90°$ . . . line from midpoint of circle to midpoint of chord is ⊥ on chord. In △PTS and △OAS: $\hat{T} = \hat{A}_2 = 90°$ . . . proven $\hat{S}_1 = \hat{S}_2$ . . . given $\therefore \hat{P}_1 = \hat{O}_2$ . . . int ∠'s △'s ∴ △PTS ||| △OAS . . . A;A;A	✓ S&R ✓ S&R ✓ S&R ✓ R
11.2.3	Prove that $TS.OP = QA.PS$	(4)
	$\frac{TS}{AS} = \frac{PS}{OS}$ . . . $\Delta PTS \|\|\|\Delta OAS$ But QA = AS . . . given And OS = OP . . . radii $\therefore \frac{TS}{QA} = \frac{PS}{OP}$ $\therefore TS\,.\,OP = QA\,.\,PS$	✓ S & R ✓ S ✓ S & R ✓ Ratio (4)
		[16]
		GRAND TOTAL: [150]