EXAMINATION	NATIONAL SENIOR CERTIFICATE
GRADE	12
DATE	NOVEMBER 2024
SUBJECT	MATHEMATICS
PAPER	2
MARK TOTAL	150
DURATION (HOURS)	3
NUMBER OF PAGES	32

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INSTRUCTIONS

Read the following instructions carefully before answering the questions.

1. This paper consists of 11 questions. Answer all the questions.
2. Clearly show ALL calculations, diagrams, graphs etc. that you have used in determining your answers.
3. Answers only will not necessarily be rewarded full marks.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are not necessarily drawn to scale.
7. An information sheet with formulae is included at the end of the question paper.
8. Write neatly and legibly.
9. Answer all the questions **on the exam paper** on the lines provided after each question.
10. Additional writing space is provided at the end of the paper. Clearly indicate if you use the additional writing space to complete a question

QUESTION 1

Pamela recorded the amount of data in MB that she used on each of the first 15 days in May. The information is shown in the table below:

26	13	3	18	12	34	24	58	16	10	15	69	20	17	40

1.1 Calculate the mean for the data set. (1)

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1.2 Determine the standard deviation for the data set. (1)

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1.3 Determine the number of days for which the data used was greater than one standard deviation above the mean. (2)

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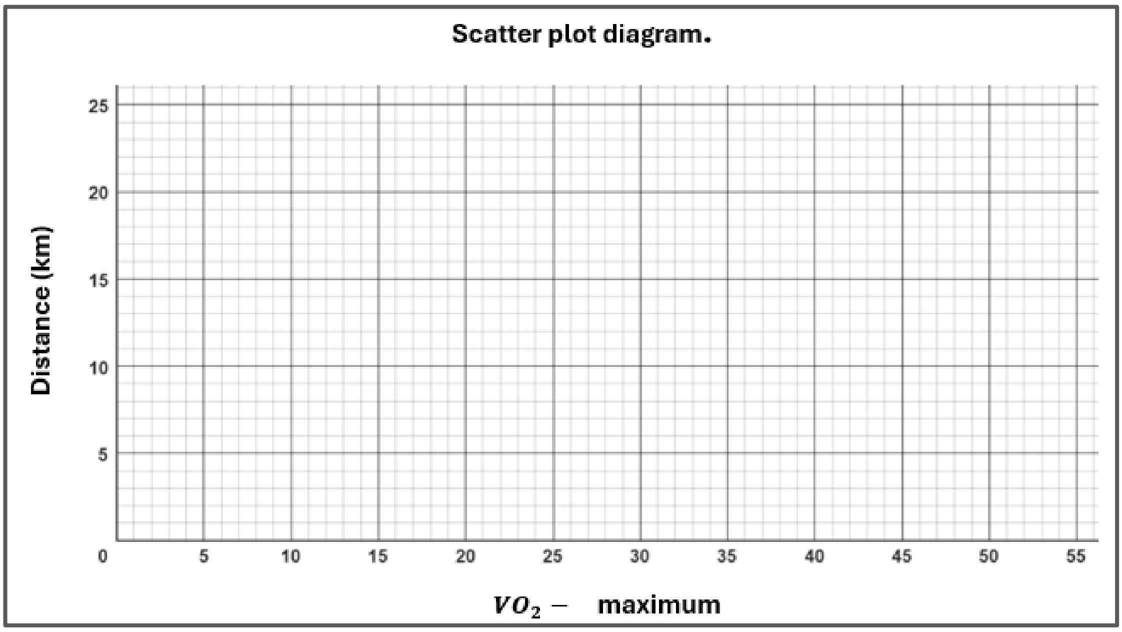
[4]

QUESTION 2

An athlete's ability to take in and use oxygen is known as their VO_2-maximum.
The table below shows eleven athletes' VO_2 maximum and the distance they ran in an hour.

VO_2-max	50	55	20	30	40	25	30	50	40	35	35
Distance (km)	15	18	8	13	14	10	12	16	11	13	5

2.1 Draw a scatter plot of the above data on the set of axes below. (2)



2.2 Calculate the equation of the least squares regression line of the data. (3)

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2.3 Draw the least square regression line on the scatter plot drawn in **QUESTION 2.1** above. (3)

2.4 Identify if there are any outliers in the data. Provide a reason for your answer. (2)

2.5 Predict the VO_2 maximum of an athlete who ran 19 km. (2)

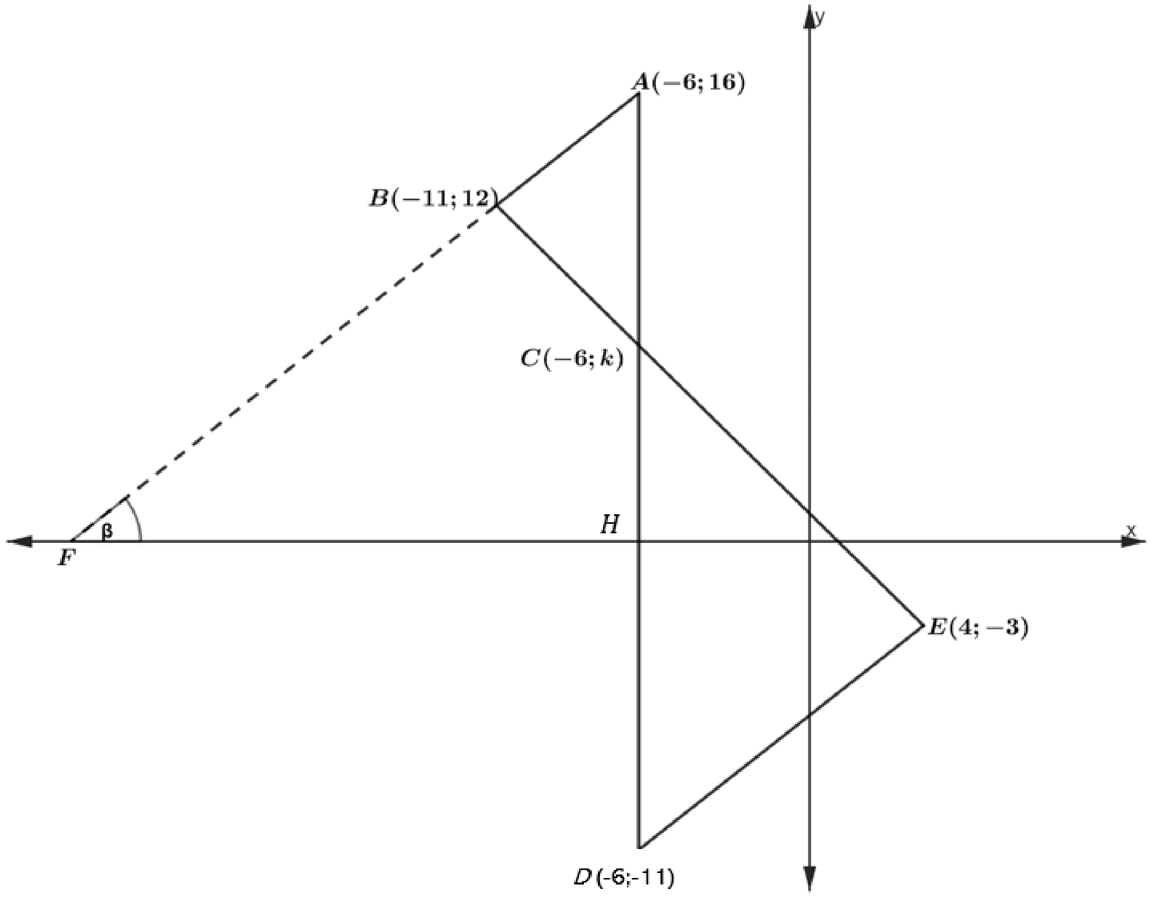
2.6 Determine the correlation coefficient of the data and comment on the correlation. (2)

[14]

QUESTION 3

In the diagram below:

- $A(-6;16)$; $B(-11;12)$; $D(-6;-11)$ and $E(4;-3)$ are given.
- AD intersects BE at point $C(-6;k)$.
- AB produced meets the x-axis in F.
- $H\hat{F}A = \beta$.
- AD intersects the x-axis at H.
- ACD and BCE are straight lines.



3.1 Show that the value of $k = 7$. (4)

3.2 Calculate the length of BC. (2)

3.3 Show that C is **NOT** the midpoint of BE. (2)

3.4 Calculate the size of $B\hat{A}D$. (5)

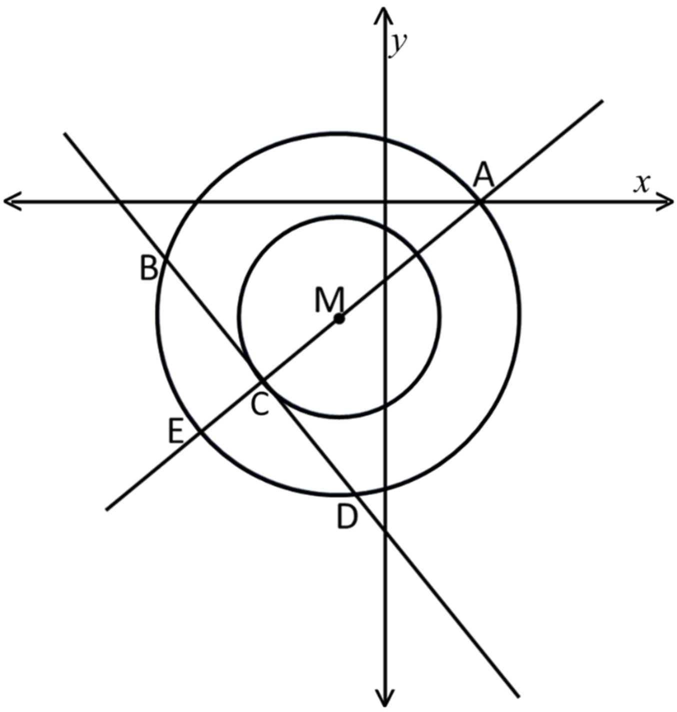
3.5 Calculate the numerical value of $\frac{Area\ of\ \Delta ABC}{Area\ of\ \Delta DEC}$ (5)

[18]

QUESTION 4

In the diagram below:

- The small circle (P_1) and the big circle (P_2) have the same center M.
- A and E are points on circle P_2.
- EM intersects circle P_1 at point C.
- The tangent BD to circle P_1 intersects circle P_2 at B and D.
- The equation of circle P_1 is given by $x^2 + 2x + y^2 + 10y + 6 = 0$.
- The equation of line EM is $y = x - 4$.



4.1 Give the coordinates of centre M. (3)

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4.2 Determine the radius of circle P_1. (1)

4.3 C is the point where line EM and circle P_1 intersects. Determine the coordinates of C, correct to two decimal places. Show all workings. (6)

4.4 Determine the gradient of the tangent to circle P_2 at point E. (3)

4.5 Determine MD, a radius of circle P_2. (4)

4.6 Write down the equation of P_2 in the form $(x-a)^2+(y-b)^2=r^2$. (1)

4.7 Prove that point $q(\sqrt{5}\,;0)$ lies inside circle P_2. Show all your calculations. (4)

[22]

QUESTION 5

5.1 If $7\cos x = 3$ and $x \epsilon [90°\,; 360°]$, calculate the value of $3\cos 2x$ **without the use of a calculator**. (4)

5.2 Prove that $2\sin x - \cos^2 x$ can be written as $(\sin x + 1)^2 - 2$. (3)

5.3

5.3.1 Determine the general solution for the value(s) of x if:

$$5\sin 2x - \cos 2x = 0$$ (5)

5.3.2 For which value(s) of x will $5\tan 2x - 1 = 0$ where $x\epsilon[-180°; 180°]$ be undefined? (2)

5.4 Simplify to a single trigonometric function, **without the use of a calculator**:

$$-1 + \cos(180° - \theta).\sin(\theta - 90°)$$ (3)

5.5 If $\cos 44° = \sqrt{p}$, determine $\sin^2 68°$ in terms of p, **without the use of a calculator.** (4)

5.6 Prove the following identity:

$$\frac{\sin 7x \cdot \cos 5x - \cos 7x \cdot \sin 5x}{\tan 2x} - 1 = -2\sin^2 x$$ (4)

5.7 Given: $\tan x = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + \ldots}}}}$

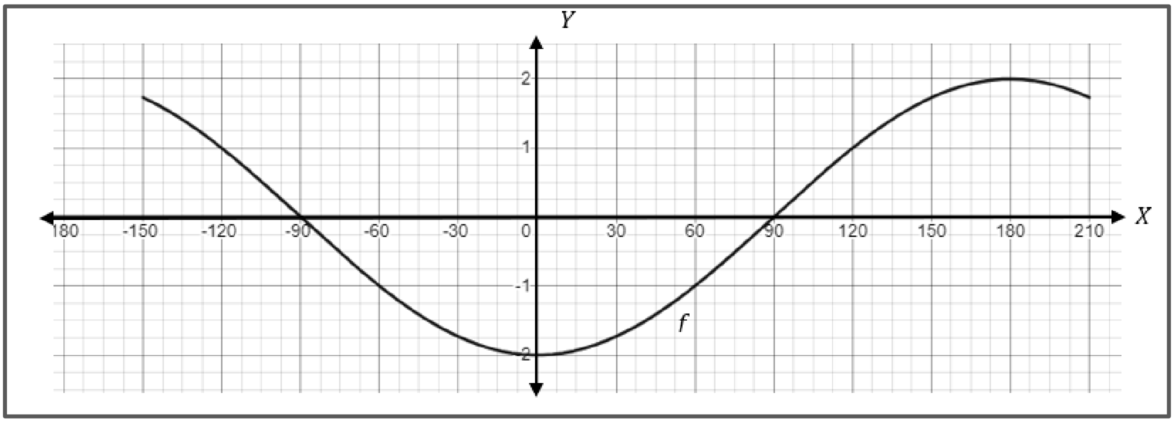
Prove that: $\sin x = \sqrt{\cos x(\cos^2 x + \sin x)}$ (5)

[30]

QUESTION 6

6.1 Determine the value(s) of x **without the use of a calculator**, if $\sin(x - 30°) + 2\cos x = 0; x \epsilon [-150°; 210°]$ and $\cos x \neq 0$. (6)

6.2 The diagram below shows the graph of $f(x) = -2\cos x$, where $x \epsilon [-150°; 210°]$.



6.2.1 Sketch the graph of $g(x) = \sin(x - 30°)$ on the same diagram given above. Clearly show all intercepts with the axes as well as the turning points. (4)

6.2.2 Write down the amplitude of f . (1)

6.3 Use your graphs to determine the value(s) of x, in the interval $x\epsilon[-150°; 210°]$ such that:

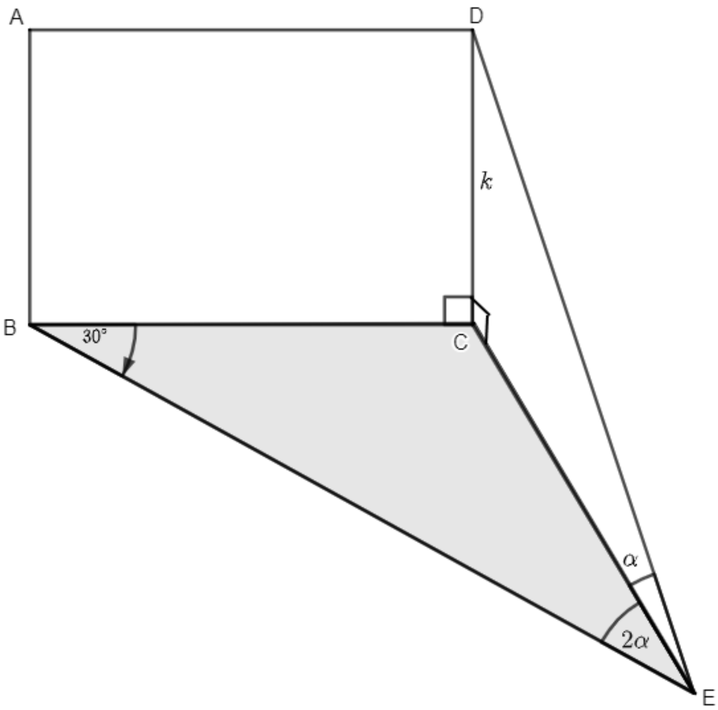
6.3.1 $g(x) - f(x) = 0$. (2)

6.3.2 Indicate, using the symbols A and B, your answer to question 6.3.1 on the graph drawn in question 6.2.1. (1)

[14]

QUESTION 7

In the diagram B, C and E lie in the same horizontal plane. ABCD is a rectangular cardboard piece and CDE is a triangular cardboard piece with a right angle at C, and $DC = k$. The cardboard pieces are placed perpendicular to the horizontal plane as shown in the diagram. The angle of elevation from E to D is α. $C\hat{E}B = 2\alpha$ and $E\hat{B}C = 30°$.



7.1 Write CE in terms of k and α. (2)

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7.2 Prove that $BC = 4\,k\,\cos^2\alpha$ (4)

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7.3 Show that the area of △BCE is equal to:

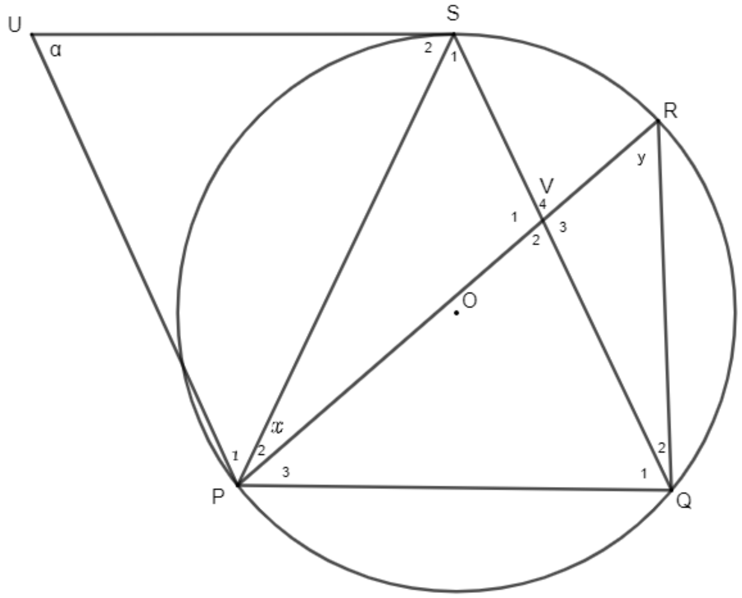
$$\frac{2k^2 \cos^3 \alpha \, . \sin(30^\circ + 2\alpha)}{\sin \alpha}$$ (3)

[9]

QUESTION 8

In the diagram:

- O is the centre of the circle.
- The circle passes through P, Q, R, and S.
- SQ and PR intersect at V.
- SU is a tangent to the circle at S.
- UP is drawn.
- $\hat{U} = \alpha$.
- $\hat{R} = y$.
- $\hat{P}_2 = x$.
- $\alpha = x + y$.



8.1 Give, with a reason the size of:

8.1.1 $\hat{Q}_2$ (2)

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8.1.2 $\hat{S}_1$ (1)

8.2 Prove that VSUP is a cyclic quadrilateral. (3)

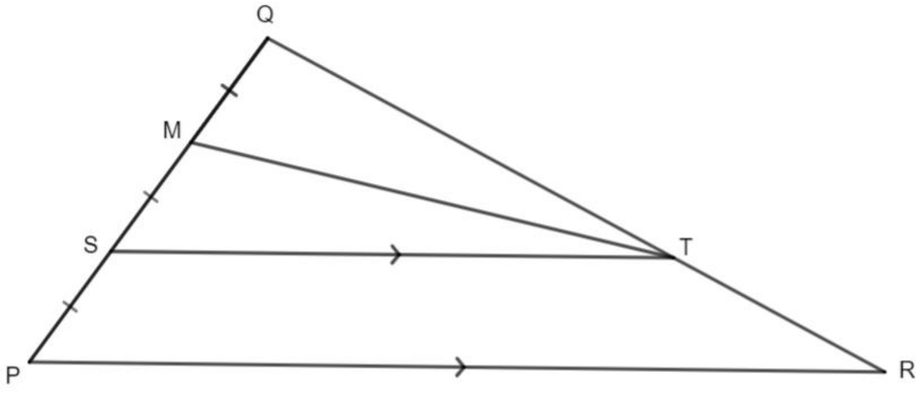
8.3 If $PQ = QR$, prove that PQ is a tangent to the circle through V, S, U and P. (3)

[9]

QUESTION 9

In the diagram △PQR is drawn with S and T points on PQ and QR respectively.

- M is a point on QS.
- ST ∥ PR
- PS = SM = MQ



9.1 Let $SP = k$. Calculate $\frac{TR}{QT}$. (2)

9.2 Determine the numerical value of $\frac{Area\ of\ \Delta QMT}{Area\ of\ \Delta QPR}$. (4)

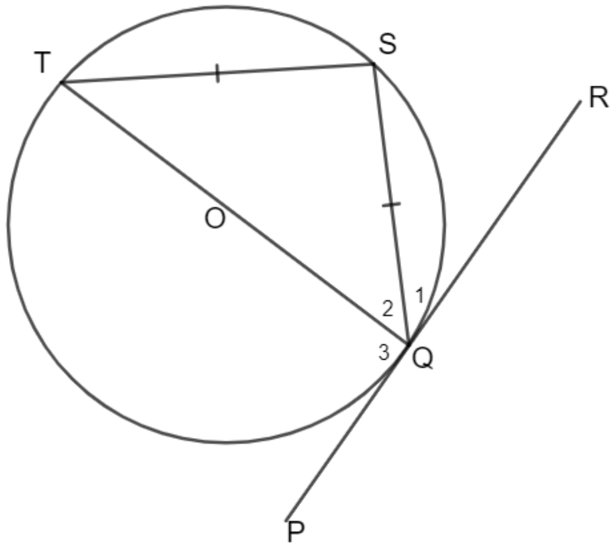
9.3 If ST = 22 cm, determine the length of PR. (4)

[10]

QUESTION 10

In the diagram:

- TOQ is a diameter of circle with centre O.
- T, S and Q lie on the circumference of the circle.
- PR is a tangent to the circle at Q.
- TS = SQ.

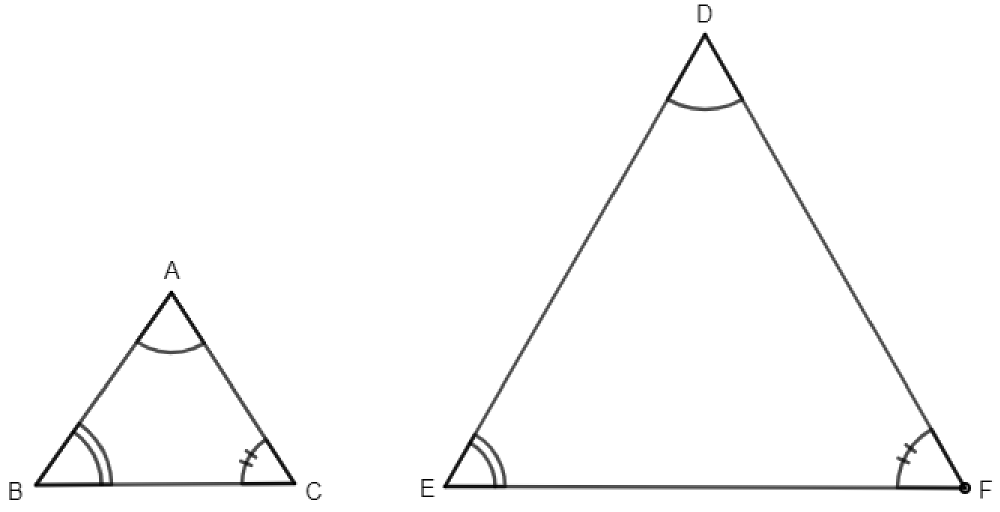


Determine, with reasons, the size of $\hat{Q}_1$. (4)

[4]

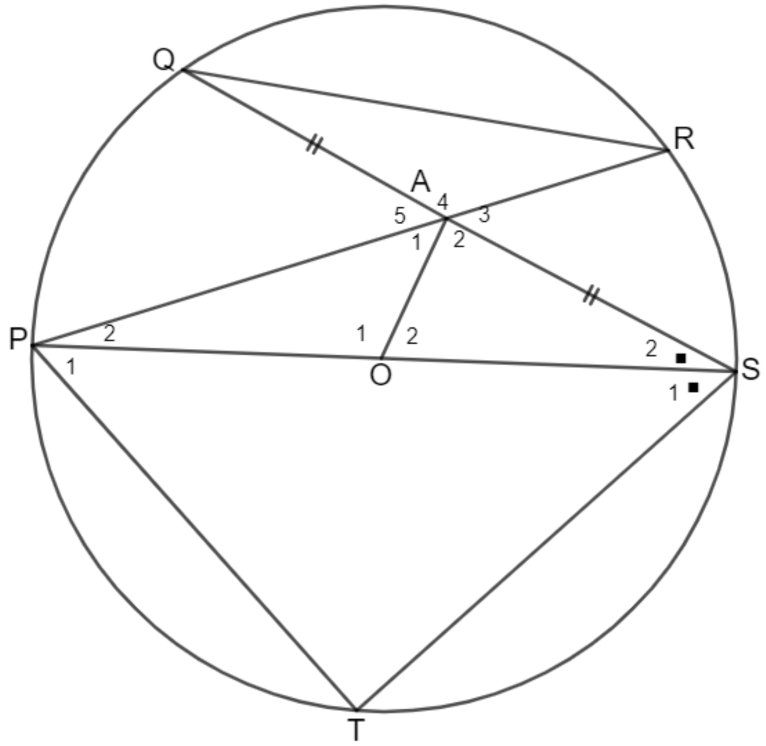
QUESTION 11

11.1 In the diagram ΔABC and ΔDEF is given with $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$.
Use this diagram to prove that $\frac{AB}{ED} = \frac{BC}{EF}$. (6)



11.2 In the diagram:

- POS is the diameter of the circle with centre O.
- QAS and PAR are straight lines.
- P, Q, R, S and T lie on the circumference of the circle.
- $QA = AS$.
- $\hat{S}_1 = \hat{S}_2$.
- SA produced meets the circle at Q.
- QR, PT and ST are drawn.



11.2.1 Write down, with a reason, the size of $\hat{T}$. (2)

11.2.2 Prove that $\triangle PTS\ |||\ \triangle OAS$. (4)

11.2.3 Prove that $TS.OP = QA.PS$ (4)

[16]

GRAND TOTAL: [150]

ADDITIONAL WRITING SPACE

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $\quad r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\quad$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$