MARKING GUIDELINES

EXAMINATION	NATIONAL SENIOR CERTIFICATE
GRADE	12
DATE	NOVEMBER 2025
SUBJECT	MATHEMATICS
PAPER	2
MARK TOTAL	150
DURATION (HOURS)	3
NUMBER OF PAGES	28

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SOUTH AFRICAN COMPREHENSIVE ASSESSMENT INSTITUTE

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FINAL APPROVED MARKING GUIDELINES	
DATE OF MEETING	
UMALUSI MODERATOR	
CHIEF MARKER	
INTERNAL MODERATOR	

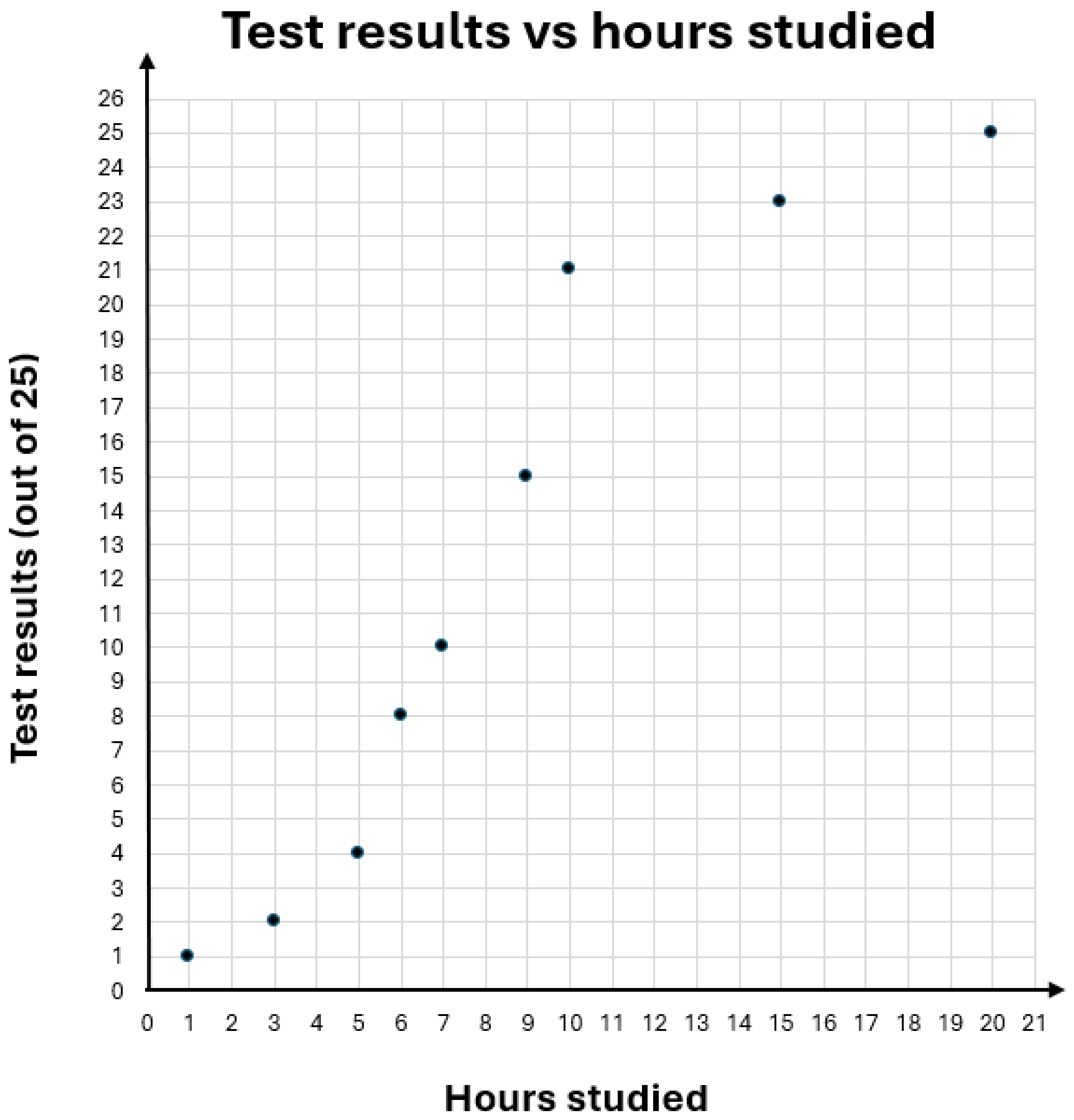
NOTE:

- If a candidate answered a question **TWICE**, only mark the first attempt.
- If a candidate cancelled the first attempt and did not answer the question again, mark the cancelled attempt.
- The principle of consistent accuracy **(CA)** will be applied throughout the memorandum.
- If a candidate has given the answer only, without showing any necessary working out, only **ONE** mark will be given.

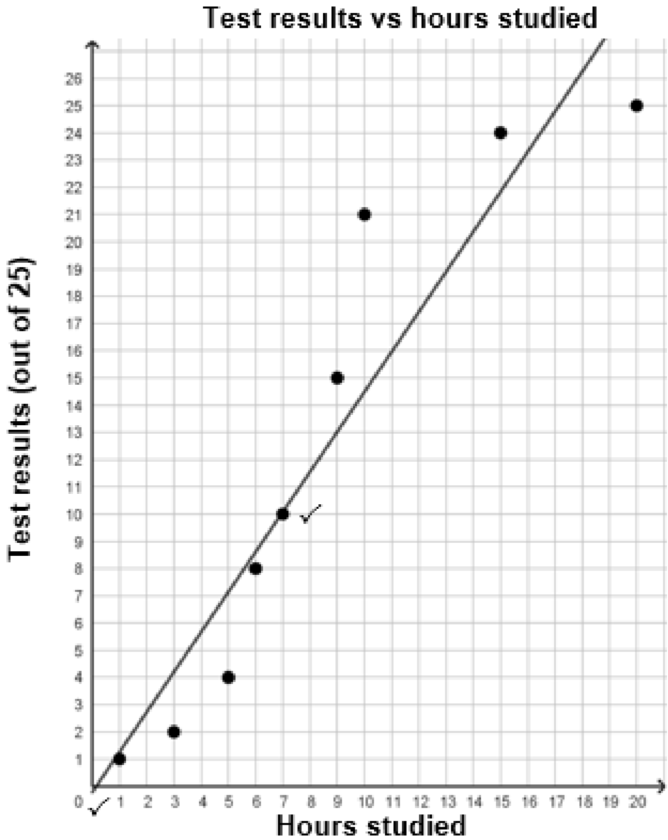
QUESTION 1 [20]

1.1 The following scatter plot shows the hours studied (x) for a Mathematics test and the corresponding test scores (y), out of 25, for nine grade 12 students. The data for the scatter plot is obtained from the following table:

Hours studied (x)	1	3	5	6	7	9	10	15	20
Test results (/25) (y)	1	2	4	8	10	15	21	23	25



1.1.1	Calculate the mean of the test scores. $\bar{y} = 12{,}11$	✓ 12,11	(1)

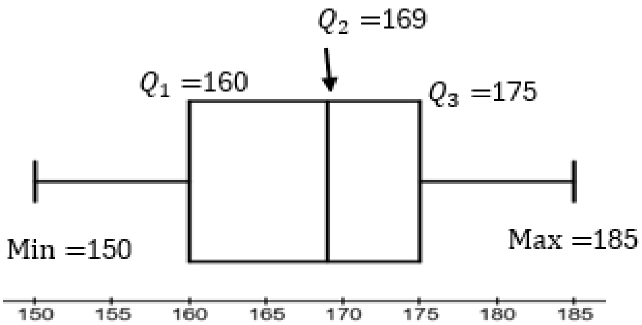
1.1.2	Determine the equation of least squares regression line for the data. $A = -0{,}21$ $B = 1{,}46$ $y = 1{,}46x - 0{,}21$	✓ Values of A AND B ✓ $y = 1{,}47x + 1{,}24$	(2)
1.1.3	Draw the least square regression line on the scatter plot provided **IN QUESTION1.1**. 	✓ y- intercept ✓ Passing through (7,10)	(2)
1.1.4	Predict the test score for a student who studies 14,5 hours per week. $y = 1{,}46x - 0{,}21$ $y = 1{,}46(14{,}5) - 0{,}21$ $y = 20{,}96$ $y = 21$	✓ $1{,}46(14{,}5) - 0{,}21$ ✓ 21 Answer only full marks	(2)
1.1.5	Determine the correlation coefficient for the data and comment on the correlation. $r = 0{,}94$ Strong positive correlation.	✓ 0,94 ✓ Strong, positive	(2)

1.1.6	**Calculate** whether there are any outliers in the test scores. Q_1=3 Q_3=22 $\text{IQR} = 22 - 3 = 19$ Lower limit $= 3 - 1{,}5 \times 19 = -25{,}5$ Upper limit $= 22 + 1{,}5 \times 19 = 50{,}5$ There are NO (zero) outliers in the test scores.	✓ IQR ✓ Lower limit ✓ Upper limit ✓ NO (zero)	(4)
1.1.7	Explain any potential risks or inaccuracies in extrapolating the data beyond the given range. The maximum marks for the test is 25, therefore extrapolating the data will not show a higher test mark, even though more hours have been spent studying. The test marks remain constant after 25 marks. (plateau in the trend). **OR** Extrapolating data assumes that the student will receive a higher mark, but it does not consider that the study time of 25 hours may be ineffective. Student may have fatigue or learning problems. **OR** Extrapolation modelled strictly on the least square regression line will have a potential overestimation for marks higher than 25, or underestimation for marks lower than 0, which will not be possible.	✓ Explaining max for test results ✓ Explaining plateau effect / unrealistic values	(2)

1.2	The heights of 10 students in grade 11 is recorded as per the table below:		

Heights of students (cm)	150	155	160	165	168	170	172	175	180	185

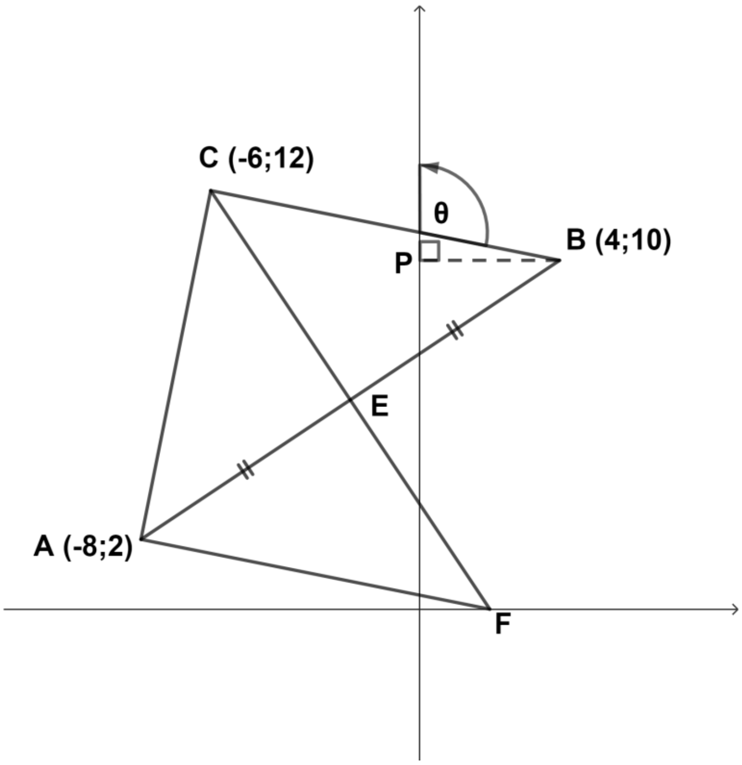
1.2.1	Draw a box-and-whisker diagram of the data.		
	Min = 150 Q_1 = 160 Median = 169 Q_3 = 175 Max = 185	✓ Min & Max ✓ Q_1 & Q_3 ✓ Median ✓ Scale	(4)



1.2.2	Comment on the skewness of the data. Skewed to the left.	✓ Left	(1)
			[20]

QUESTION 2 [19]

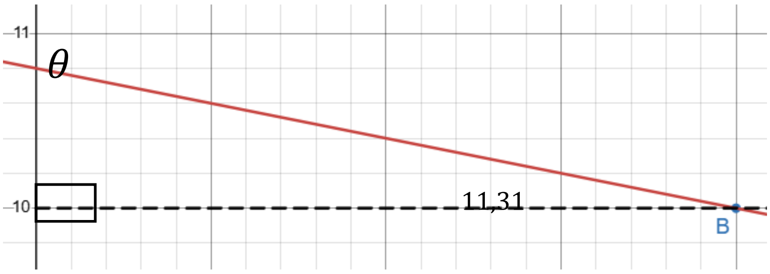
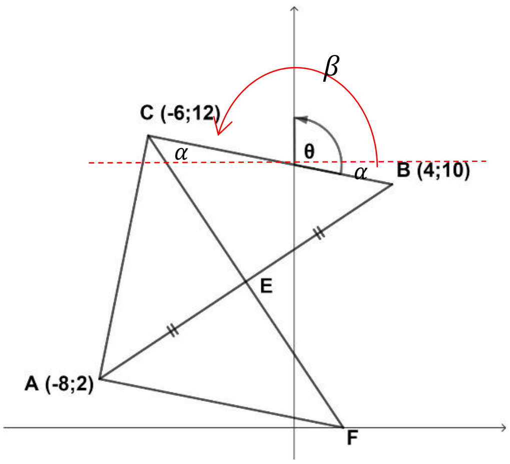
In the diagram $A(-8;2)$, $B(4;10)$ and $C(-6;12)$ are given. E is the midpoi the x –intercept of line CE extended. AF is drawn. The angle between BC and the y-axis equals θ.

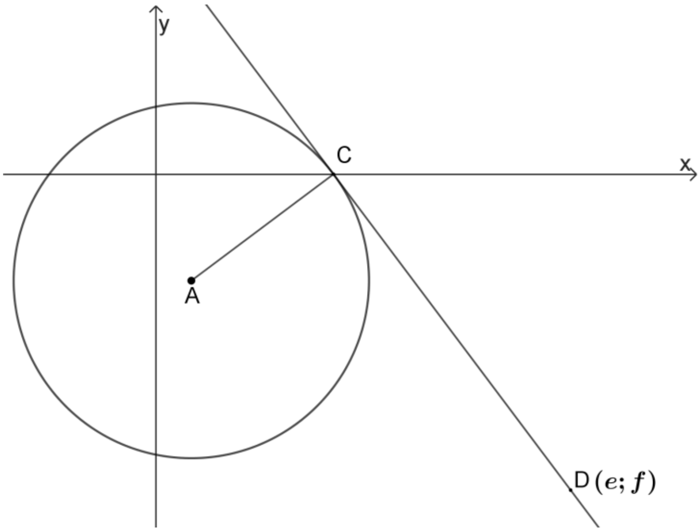


2.1	Determine the coordinates of E, the midpoint of AB. $E = \left(\frac{-8+4}{2}\ ;\ \frac{2+10}{2}\right)$ $E = (-2;6)$	✓ Subst in correct formula ✓ (-2;6)	(2)
2.2	Show that CE ⊥ AB. $m_{CE} = \frac{12-6}{-6+2}$ $m_{CE} = \frac{6}{-4}$ $m_{CE} = -\frac{3}{2}$ $m_{AB} = \frac{10-2}{4+8}$ $m_{AB} = \frac{8}{12}$ $m_{AB} = \frac{2}{3}$ $\therefore m_{CE} \times m_{AB} = -\frac{3}{2} \times \frac{2}{3}$ $m_{CE} \times m_{AB} = -1$ ∴ CE ⊥ AB	✓ m_{CE} ✓ m_{AB} ✓ $m_{CE} \times m_{AB} = -\frac{3}{2} \times \frac{2}{3} = -1$	(3)

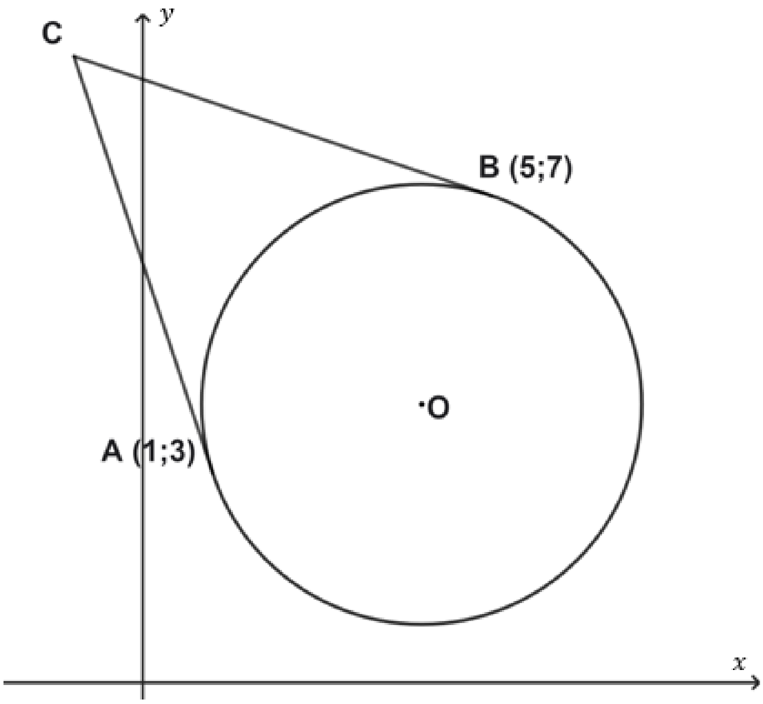
2.3	Determine the equation of CE. Equation of CE: E(-2;6) $m_{CE} = -\frac{3}{2}$ (from 2.1.2) $y = mx + c$ OR $y - y_1 = m(x - x_1)$ $6 = -\frac{3}{2}(-2) + c$ $y - 6 = -\frac{3}{2}(x + 2)$ $c = 3$ $y = -\frac{3}{2}x + 3$	✓ Subst. (-2;6) ✓ $y = -\frac{3}{2}x + 3$	(2)
2.4	Determine the coordinates of F. $-\frac{3}{2}x + 3 = 0$ $-\frac{3}{2}x = -3$ $x = 2$ F(2; 0)	✓ Coordinates	(1)
2.5	Show that BE = EF. $BE = \sqrt{(4+2)^2 + (10-6)^2}$ $BE = 2\sqrt{13}$ units $EF = \sqrt{(-2-2)^2 + (6-0)^2}$ $EF = 2\sqrt{13}$ units	✓ Subst. in distance formula ✓BE= $2\sqrt{13}$ ✓ Subst. in distance formula EF=$2\sqrt{13}$	(3)

2.6	D is a point in the first quadrant such that BDFE is a square. Determine the coordinates of D. Using translation: $E(-2;6)$ and $F(2;0)$ $D(4+4;10-6)$ ✓ ✓ $D(8;4)$ ✓ **OR** $\frac{y-0}{x-2}=\frac{10-6}{4+2}$ $3y=2x-4$ and $\frac{y-10}{x-4}=-\frac{6}{4}$ $4y=6x-64$ Solve simultaneously : $x=8$ and $y=4$ $D(8;4)$ ✓ ✓✓ **OR** $EF=BD=EB=FD=2\sqrt{13}$ units $FD^2=(x-2)^2+(y)^2=52$ $x^2-4x+4+y^2=52$ . . . ① $m_{FD}=\frac{y-0}{x-2}=-\frac{10-6}{4+2}$ . . . ② $6y=4x-8$ $3y=2x-4$ $y=\frac{2}{3}x-\frac{4}{3}$ $x^2-4x+4+\left(\frac{2}{3}x-\frac{4}{3}\right)^2=52$ $x^2-4x+4+\frac{4}{9}x^2-\frac{16}{9}x+\frac{16}{9}-52=0$ $\frac{13}{9}x^2-\frac{52}{9}x-\frac{416}{9}=0$ $13x^2-52x-416=0$ $x=8$ or $x\neq-4$ $\therefore y=4$ D(8;4) ✓ ✓✓	✓ (x+4) ✓ (y-6) ✓ Coordinate **OR** ✓ 2x equations ✓ Method ✓ Coordinates **OR** ✓ 2x equations ✓ Method ✓ Coordinates	(3)

2.7	Determine the value of θ  $m_{BC} = \frac{10-12}{4+6}$ ✓ $= -\frac{1}{5}$ ✓ $\tan C\hat{B}D = \frac{1}{5}$ $C\hat{B}D = 11{,}31°$ ✓ $\theta = 90° + 11{,}31°$ ✓ $= 101{,}31°$ ✓ **OR**  $\tan\beta = \frac{10-12}{4+6}$ ✓ $= -\frac{1}{5}$ ✓ $\beta = 168{,}69°$ ✓ $\alpha = 11{,}31°$ ✓. . . ∠'s on str line and vertically opposite $\theta = 90° + 11{,}31° = 101{,}31°$ ✓	✓ m_{BC} correct formula substitution ✓ $\tan C\hat{B}D = \frac{1}{5}$ ✓ $\tan C\hat{B}D = 11{,}31°$ ✓ $\theta = 90° + 11{,}31°$ ✓ Answer **OR** ✓ m_{BC} correct formula substitution ✓ $\tan C\hat{B}D = \frac{1}{5}$ ✓ $\beta = 168{,}69°$ ✓ $\alpha = 11{,}31°$ ✓ Answer	(5)
			[19]

QUESTION 3			[21]
3.1	The diagram below shows a circle with centre A and equation $(x-1)^2+(y+3)^2=25$. C is an x-intercept of the circle. A tangent to the circle at C, CD, is drawn with $D(e;f)$. 		
3.1.1	Write down the coordinates of A and the length of AC, the radius of circle A. A $(1;-3)$ $AC = 5$ units	✓ $A(1;-3)$ ✓ $AC = 5$	(2)
3.1.2	Determine the equation of the tangent CD. $m_{AC} = \frac{-3-0}{1-5}$ $m_{AC} = \frac{3}{4}$ $m_{AC} \times m_{CD} = -1$ radius ⊥ tangent $m_{CD} = -\frac{4}{3}$ $y = mx + c$ OR $y - y_1 = m(x - x_1)$ $0 = -\frac{4}{3}(5) + c$ $y - 0 = -\frac{4}{3}(x-5)$ $c = \frac{20}{3}$ $y = -\frac{4}{3}x + \frac{20}{3}$	✓ m AC ✓ mCD ✓ Subst. (5;0) ✓ Equation	(4)

3.1.3	The length of CD is $\frac{25}{3}$ units. Calculate the values of e and f. D lies on CD: $f=-\frac{4}{3}e+\frac{20}{3}$ $CD^2=\left(\frac{25}{3}\right)^2=(e-5)^2+\left(-\frac{4}{3}e+\frac{20}{3}-0\right)^2$ $\frac{625}{9}=e^2-10e+25+\frac{16}{9}e^2-\frac{160}{9}e+\frac{400}{9}$ $625=25e^2-250e+625$ $25e(e-10)=0$ $e=10$ $f=-\frac{20}{3}$	✓ $f=-\frac{4}{3}e+\frac{20}{3}$ ✓CD distance formula subst. ✓Simplify ✓ Std form ✓ Factors ✓ Both answers	(6)

3.2	A circle, with equation $(x-4)^2 + (y-4)^2 = 10$, passes through the points $A(1;3)$ and $B(5;7)$. Tangent CA and tangent CB are drawn from point C as shown in the diagram.  Another circle is drawn with centre at point C and radius $\sqrt{10}$. Determine if the two circles will intersect, or not. Justify your answer.	✓ (5;3) ✓ $d = 7{,}211$ ✓ $r_1 + r_2 = 8$ ✓ $8 > 7{,}211$ ✓ Conclusion	(9)
	$\text{OB} = \text{OA} = \sqrt{10}$ units Radii of circle $m_{AO} = \frac{3-4}{1-4} = \frac{1}{3}$ $m_{AC} = -3$ ✓ AC: $y - 3 = -3(x-1)$ $y = -3x + 6$ . . . ① ✓ $m_{BO} = \frac{7-4}{5-4} = 3$ $m_{BC} = -\frac{1}{3}$ ✓ BC: $y - 7 = -\frac{1}{3}(x-5)$ $y = -\frac{1}{3}x + \frac{26}{3}$ . . . ② ✓ Point C: $-3x + 6 = -\frac{1}{3}x + \frac{26}{3}$ $x = -1$ $y = -3(-1) + 6 = 9$	✓ $m_{AC} = -3$ ✓ Eq. AC ✓ $M_{BC} = -\frac{1}{3}$ ✓ Eq. BC ✓ C (-1;9) ✓ OC = 7,07 ✓ $r_O + r_C = 6{,}32$ ✓ $OC > r_O + r_C$ ✓ do not intersect	

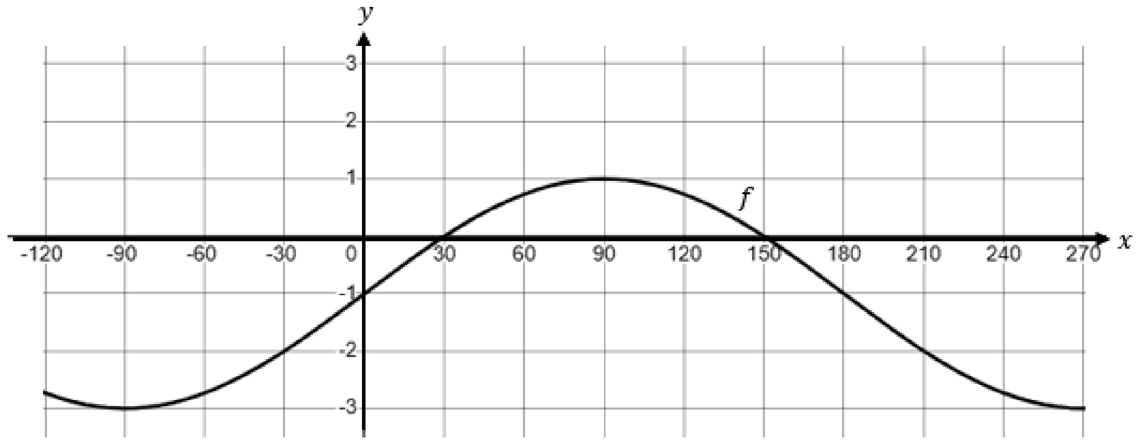
	$C(-1;9)$ ✓ Eq. with C as centre: $(x+1)^2+(y-9)^2=10$ Distance $OC=\sqrt{(4+1)^2+(4-9)^2}$ $=5\sqrt{2}\approx 7{,}07$ units ✓ $r_O+r_C=\sqrt{10}+\sqrt{10}=2\sqrt{10}\approx 6{,}32$ units ✓ $OC>r_O+r_C$ ✓ ∴ circles **do not** intersect. ✓		(9)
			[21]

QUESTION 4			**[12]**
4.1	If $\sin 21°.\cos 21° = p$, determine, **without using a calculator,** the following in terms of p. Leave answers in surd form if necessary.		
4.1.1	$\sin 42°$ $= \sin 2(21°)$ $= 2\sin 21° \cos 21°$ $= 2p$	✓ 2 sin 21°cos21° ✓ 2p	(2)
4.1.2	$2\sin^2 21° - 1$ $2\sin^2 21° - 1 = -\cos 42°$ $2\sin 21°.\cos 21° = \sin 42°$ $\sin 42° = 2p$ $\cos^2 42° + \sin^2 42° = 1$ $\cos^2 42° = 1 - \sin^2 42°$ $= 1 - (2p)^2$ $= 1 - 4p^2$ $\cos 42° = \sqrt{1-4p^2}$ $2\sin^2 21° - 1 = -\sqrt{1-4p^2}$ $\sin^2 21° = \frac{1-\sqrt{1-4p^2}}{2}$	✓ $-\cos 42°$ ✓ $\cos^2 42° = 1 - \sin^2 42°$ ✓ $= -\sqrt{1-4p^2}$ ✓ Answer	(4)
4.2.	**Without using a calculator**, simplify the following expression: $\frac{\sin 290°}{\tan 215° \,.\cos(-35°)\,.\sin 55°}$		
	$= \frac{\sin(360° - 70°)}{\tan(180° + 35°).\cos 35°\,.\sin(90° - 35°)}$ $= \frac{-\sin 70°}{(\tan 35°).\cos 35°\,.\cos 35°}$ $= \frac{-2\sin 2(35°)}{\frac{\sin 35°}{\cos 35°} \times \cos 35°\,.\cos 35°}$ $= \frac{-2\sin 35° \cos 35°}{\sin 35° \cos 35°}$ $= -2$	✓ $-\sin 70°$ ✓ tan 35° ✓ cos 35° ✓ -2 sin 35° x cos 35° ✓ $\frac{\sin 35°}{\cos 35°}$ ✓ -2	(6)
			[12]

QUESTION 5			**[11]**
5.1	Prove the following identity: $$\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = 1 + \sin x \cos x$$		
	$\text{LHS} = \frac{(\cos x - \sin x)(\cos^2 x + \cos x \sin x + \sin^2 x)}{(\cos x - \sin x)}$ $= 1 + \cos x \sin x$ $= \text{RHS}$	✓ $\cos x - \sin x$ ✓ $\cos^2 x + \cos x \sin x + \sin^2 x$ ✓ 1	(3)
5.2	Hence, determine the general solution of x, if : $$\frac{\cos^3 x - \sin^3 x}{\cos x - \sin x} = \frac{1}{2}$$		
	$1 + \sin x \cos x = \frac{1}{2}$ $\sin x \cos x = -\frac{1}{2}$ $2 \sin x \cos x = -1$ $\sin 2x = -1$ RA: $2x = 90°$ $\therefore 2x = 180° + 90° + 360°k$ $2x = 270° + 360°k$ $x = 135° + 180°k\ ; k \in Z$	✓ $1 + \sin x \cos x = 0{,}5$ ✓ $\sin 2x = -1$ ✓ RA 90° ✓ $x = 135° + 180°k$ (-1) if no $k \epsilon Z$	(4)
5.3	Simplify completely: $$2\cos 4x \cos 2x - \cos 6x + 2\sin^2 x$$		
	$2\cos 4x \cos 2x - \cos 6x + 2\sin^2 x$ $= 2\cos 4x \cos 2x - \cos(4x + 2x) + 2\sin^2 x$ $= 2\cos 4x \cos 2x - (\cos 4x \cos 2x - \sin 4x \sin 2x) + 2\sin^2 x$ $= \cos 4x \cos 2x + \sin 4x \sin 2x + 2\sin^2 x$ $= \cos(4x - 2x) + 2\sin^2 x$ $= \cos 2x + 2\sin^2 x$ $= 1 - 2\sin^2 x + 2\sin^2 x$ $= 1$	✓ Expansion ✓ $\cos 2x$ ✓ $1 - 2\sin^2 x$ ✓ Answer	(4)
			[11]

QUESTION 6 [10]

In the diagram below the graph of $f(x) = a\sin x - b$ for $x \in [-120°; 270°]$ is shown.



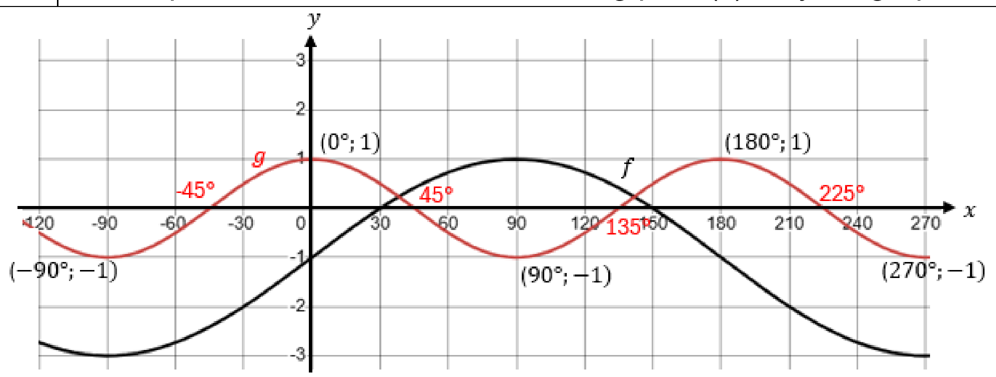
6.1 Give the values of a and b.

$a = 2$

$b = 1$

✓ a=2
✓ b=1 (2)

6.2 Sketch the graph of $g(x) = \cos 2x$ **on the same set of axes in QUESTION 6 above in your answerbook.** Clearly indicate the intercepts with the axes and the turning point(s) on your graph.



✓ 5 turning points
✓ (0;1)
✓ 4 x-values
✓ Shape (4)

6.3 Use your graph and determine the value(s) of x for which:

$f(x) - g(x) = 2$

$x = 90°$

✓ 90° (1)

6.4 Write down the period of $f(3x)$.

120°

✓ 120° (1)

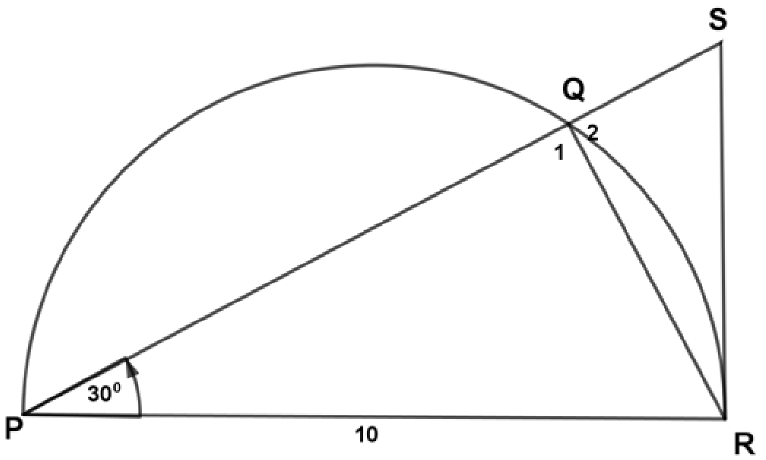
6.5 Write down the range of k if $k(x) = \frac{1}{2}g(x) + 1$.

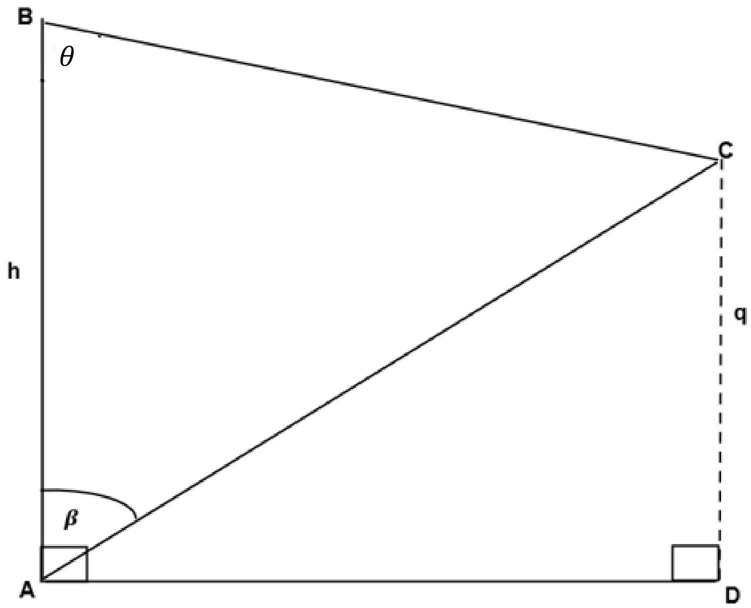
$k \in \left[\frac{1}{2}; \frac{3}{2}\right]$

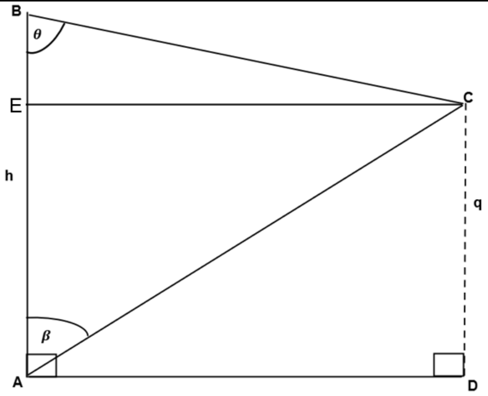
✓ Critical values
✓ Notation (2)

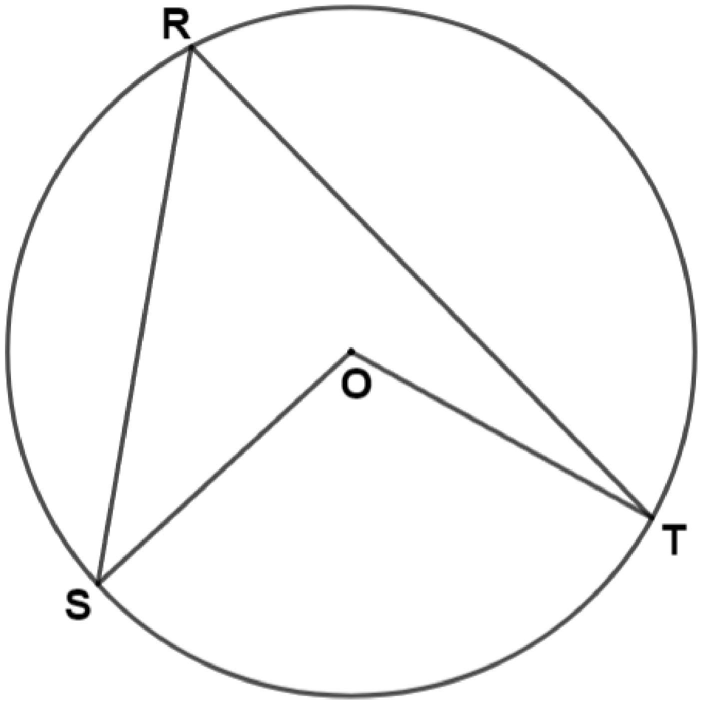
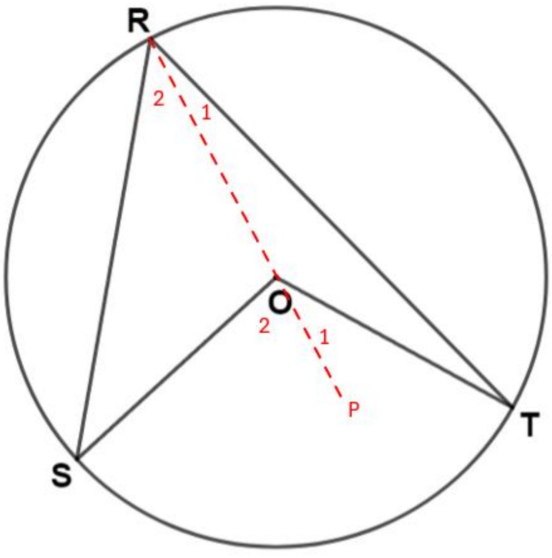
[10]

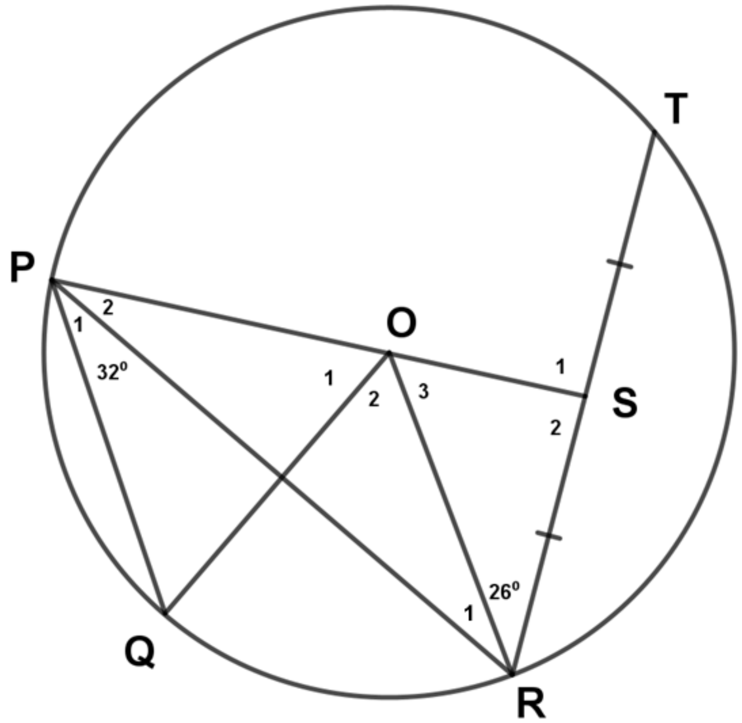
QUESTION 7 [15]

7.1	In the diagram points below, P, Q and R lie on the circumference of the semi-circle. SR is a tangent to the semi-circle at R. PQ is produced to S. QR is drawn. $S\hat{P}R = 30°$ and PR = 5 units.		
	 Determine, **without the use of a calculator**, the length of QS. Leave your answer in surd form.		
	$\hat{Q}_1 = 90°$ . . . ∠ in semi-circle $\frac{QR}{PR} = \sin 30°$ $QR = 10\left(\frac{1}{2}\right)$ $QR = 5$ units $P\hat{R}S = 90°$ . . . tangent ⊥ radius $\frac{SR}{PR} = \tan 30°$ $SR = 10\left(\frac{1}{\sqrt{3}}\right)$ $SR = \frac{10}{\sqrt{3}}$ units $QS^2 = SR^2 - QR^2$ Pyth $QS^2 = \frac{100}{3} - 25 = \frac{25}{3}$ $QS = \frac{5}{\sqrt{3}}$ $\hat{S} = 60°$ . . . Int ∠ of ΔSPR $\frac{QS}{SR} = \cos 60°$ $QS = \frac{10}{\sqrt{3}}\left(\frac{1}{2}\right)$ $QS = \frac{10}{2\sqrt{3}}$ $QS = \frac{5}{\sqrt{3}}$ units	✓ $\hat{Q}_1 = 90°$ & reason ✓ Sin ratio ✓ QR ✓ $P\hat{R}S = 90°$ & reason ✓ Tan ratio ✓ SR ✓ $QS = \frac{5}{\sqrt{3}}$	

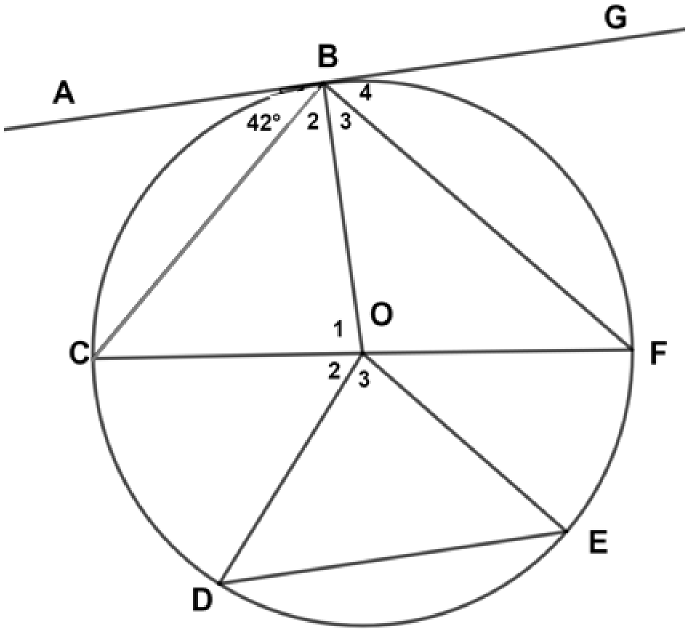
	OR Pyth: $QS^2 = RS^2 - QR^2$ $QS^2 = \frac{100}{3} - 25 = \frac{25}{3}$ $QS = \frac{5}{\sqrt{3}}$		(7)
7.2	An engineer is constructing a triangular support structure for a building, consisting of three steel beams as shown in the diagram. • Point A is located on the ground, with B positioned directly above A at a vertical height of h meters. • A second beam extends from A to C, where C is at a height of q meters above the ground at point D. • The angle between beams AB and AC is β.		
			
7.2	Show that $h = q\left(1 + \frac{\tan\beta}{\tan\theta}\right)$		
	$A\hat{C}D = \beta$ . . . Alt <'s, BA ll CD $\frac{AC}{q} = \frac{1}{\cos B}$ $AC = \frac{q}{\cos B}$ In ΔABC: $B\hat{C}A = 180° - (\theta + \beta)$ . . . Int <'s Δ $\frac{h}{\sin 180° - (\theta+\beta)} = \frac{AC}{\sin\theta}$ $h = \frac{q\sin(\theta+\beta)}{\cos\beta\sin\theta}$ $h = q\frac{\sin\theta\cos\beta + \sin\beta\cos\theta}{\cos\beta\sin\theta}$ $h = q(1 + \frac{\tan\beta}{\tan\theta})$ **OR**	✓ S & R ✓ Ratio ✓ AC ✓ S & R ✓ Sin rule ✓ $h = \cdots$ ✓ sin comp <'s ✓ Split fraction	

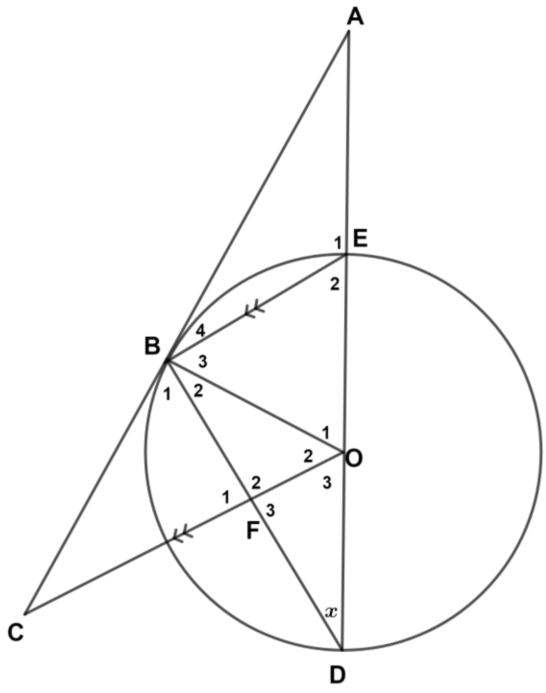
	 Construct $CE \perp BA$, CE = AD and CD = EA = q $\frac{EB}{EC} = \frac{1}{\tan\theta}$ $EB = \frac{EC}{\tan\theta}$ $\frac{AD}{q} = \tan\beta$ $AD = EC = q\tan\beta$ $\therefore EB = \frac{q\tan\beta}{\tan\theta}$ $h = BE + EA = \frac{q\tan\beta}{\tan\theta} + q$ $h = q(1 + \frac{\tan\beta}{\tan\theta})$	**OR** ✓ Construction ✓ CE=AD, CD=EA ✓ Ratio ✓ EB ✓ Ratio ✓ AD ✓ h=BE + EA ✓ Factorise	(8)
			[15]

QUESTION 8			[19]
8.1	Use the diagram below to prove the theorem that states that if an arc subtends an angle at the centre of a circle and at the circumference, then the angle at the centre is twice the size of the angle at the circumference. 		
			
	Construct RO extended to P. Let $T\hat{O}P = \hat{O}_1$ and $S\hat{O}P = \hat{O}_2$. Let $T\hat{R}O = \hat{R}_1$ and $S\hat{R}O = \hat{R}_2$ $\hat{O}_1 = \hat{R}_1 + \hat{T}$ . . . ext ∠ = sum int. opp ∠'s And $\hat{R}_1 = \hat{T}$ . . . equal radii, ∠'s opp = sides $\therefore \hat{O}_1 = 2\hat{R}_1$	✓ Construction ✓ S&R ✓ S&R ✓ S	

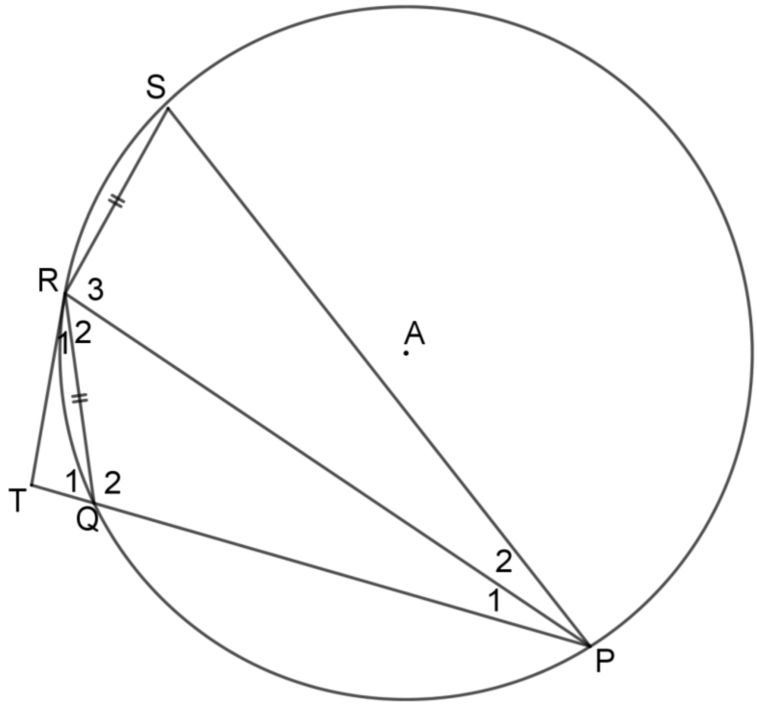
	Similarly, $\hat{O}_2 = 2\hat{R}_2$ $S\hat{O}T = \hat{O}_1 + \hat{O}_2$ $= 2\hat{R}_1 + 2\hat{R}_2$ $= 2(\hat{R}_1 + \hat{R}_2)$ $\therefore S\hat{O}T = 2S\hat{R}T$	✓ $S\hat{O}T = \hat{O}_1 + \hat{O}_2$	(5)
8.2	In the diagram below • O is the centre of the circle. • P, Q, R and T are points on the circumference of the circle. • POS bisects chord RT. • $S\hat{R}O = 26°$. • $R\hat{P}Q = 32°$. 		
	Determine, with reasons, the size of:		
8.2.1	$\hat{O}_2$ $\hat{O}_2 = 2 \times 32°$ . . . ∠ at centre = 2x∠ at circumference $\hat{O}_2 = 64°$	✓ 64° ✓ Reason	(2)
8.2.2	$\hat{O}_1$ $\hat{S}_2 = 90°$ . . . Line from midpoint bisecting chord $\hat{O}_1 + \hat{O}_2 = 116°$ . . . Ext ∠ = sum int ∠'s of ΔROS $\hat{O}_1 = 52°$	✓ S & R ✓ S & R ✓ S	

	OR $\hat{S}_2 = 90°$ . . . . Line from midpoint bisecting chord $\hat{O}_3 = 64°$ . . . Sum int ∠'s of ΔROS $\hat{O}_1 = 52°$ . . . ∠'s on straight line		(3)
8.2.3	$\hat{P}_2$ $\hat{P}_2 = \hat{R}_1$ . . . . ∠'s opp equal sides, OP = OR radii $\hat{P}_2 + \hat{R}_1 + \hat{O}_1 + \hat{O}_2 = 180°$ . . . Sum int ∠'s ΔORP $2\hat{P}_2 = 64°$ $\hat{P}_2 = 32°$ **OR** $\hat{P}_2 = \hat{R}_1$ . . . . ∠'s opp equal sides, OP = OR radii $\hat{S}_1 = \hat{P}_2 + \hat{R}_1 + 26°$ . . . . . Ext ∠ = sum int ∠'s $90° = 2\hat{P}_2 + 26°$ $\hat{P}_2 = 32°$	✓ S & R ✓ S & R ✓ Answer ✓ S & R ✓ S & R ✓ Answer	(3)

8.3	In the diagram below: • B, C, D, E and F are points on the circle with centre O. • ABG is a tangent to the circle at point B. • $A\hat{B}C = 42°$. 		
8.3.1	Calculate, with reasons, the size of $\hat{O}_1$. $\hat{F} = 42°$ . . . Tan-chord theorem $\hat{O}_1 = 84°$ . . . ∠ at midpoint = 2 x ∠ at circumference **OR** $\hat{F} = 42°$ . . . Tan-chord theorem $\hat{B}_3 = 42°$ . . . ∠'s opp = sides $\hat{O}_1 = 84°$ . . . Sum int ∠'s ΔPKN	✓ S ✓ R ✓ S ✓ R **OR** ✓ S ✓ R ✓ S & R ✓ S & R	(4)
8.3.2	If BC = DE, determine, with reasons, the size of $\hat{O}_3$. In ΔBOC and ΔDOE BC = DE given DO = BO and OE = OC radii ∴ ΔBOC ≡ ΔDOE . . . S; S; S ∴ $\hat{O}_1 = \hat{O}_3 = 84°$ **OR** $\hat{O}_1 = \hat{O}_3 = 84°$ . . . ∠'s subt by = chords at centre	✓ S & R ✓ S	(2)
			[19]

QUESTION 9			[11]
	In the diagram: • ED is the diameter of the circle with centre O. • AC is a tangent to the circle at B. • DE is extended to A. • CO intersects BD at F. • BE || CO. • $\hat{D} = x$. 		
9.1	Determine, with reasons, three angles equal to x. $\hat{B}_2 = x$ . . . ∠'s opposite = sides; BO=DO (radii) $\hat{B}_4 = x$ . . . tan-chord theorem $\hat{C} = x$ . . . corr. ∠'s; BE || CO	✓ S & R ✓ S & R ✓ S & R	(3)
9.2	Prove, with reasons, that BF = FD. $E\hat{B}D = 90°$ . . . ∠ in semi-circle $\hat{F}_3 = 90°$ . . . corr. ∠'s; BE || CO ∴ BF = FD . . . line from centre ⊥ on chord **OR** $E\hat{B}D = 90°$ . . . ∠ in semi-circle $\hat{F}_2 = 90°$ . . . co-int. ∠'s; BE || CO ∴ BF = FD . . line from centre ⊥ on chord	✓ S & R ✓ S & R ✓ R	(3)

9.3	Prove that $2\ \mathrm{DF}\,.\,\mathrm{AB} = \mathrm{AD}\,.\,\mathrm{BE}$. In ΔABE and ΔADB: $\hat{A} = \hat{A}$ . . . common ∠ $\hat{B}_4 = \hat{D}$ . . . proven (9.1) $\hat{E}_1 = D\hat{B}A$ . . . 3rd ∠ in Δ ∴ ΔABE ||| ΔADB . . . A; A; A $\frac{AD}{AB} = \frac{DB}{BE}$ . . . ΔABE ||| ΔADB $\mathrm{DB.AB} = \mathrm{BE.AD}$ But: $\mathrm{FD} = \mathrm{FB}$ . . proven $\therefore \mathrm{BD} = 2\mathrm{DF}$ $\therefore 2\ \mathrm{DF}\,.\,\mathrm{AB} = \mathrm{AD}\,.\,\mathrm{BE}$	✓ S & R ✓ S & R ✓ S ✓ Ratio ✓ S	(5)
			[11]

QUESTION 10			[12]
In the diagram below circle PQRS with centre A is drawn. QR = RS. TR is a tangent to the circle at R. Chords PQ, PR, PS, QR and RS are drawn. PQT is a straight line. 			
10.1	In ΔQRT and ΔSPR: $\hat{Q}_1 = \hat{S}$ . . . Ext ∠ cyclic quad $\hat{R}_1 = \hat{P}_1$ . . . tan-chord theorem $\hat{P}_1 = \hat{P}_2$ . . . ∠'s opp equal chords $\therefore \hat{T} = \hat{R}_3$ . . . 3rd ∠ in Δ $\therefore \Delta QRT \ \vert\vert\vert \ \Delta SPR$ . . . A ; A ; A $\frac{QR}{SP} = \frac{RT}{PR}$	✓ S & R ✓ S & R ✓ S & R ✓ S & R ✓ S & R	(5)

10.2	In ΔTPR and ΔRPS $\hat{T} = \hat{R}_3$ . . . Proven $\hat{P}_1 = \hat{P}_2$ . . . Proven $\therefore T\hat{R}P = \hat{S}$ . . . 3rd ∠ in Δ $\therefore \Delta TPR \parallel\!\mid \Delta RPS$ . . . A ; A ; A $\frac{TP}{RP} = \frac{PR}{PS}$ $PR^2 = PS.TP$ (1) $\frac{QR}{SP} = \frac{RT}{PR}$ proven $\therefore \left(\frac{PR}{SP}\right)^2 = \left(\frac{RT}{QR}\right)^2$ Use (1): $\frac{PS.TP}{SP^2} = \frac{RT^2}{QR^2}$ $\therefore \frac{PT}{PS} = \frac{RT^2}{QR^2}$ **OR** In ΔTPR and ΔRPS $\hat{T} = \hat{R}_3$ . . . Proven $\hat{P}_1 = \hat{P}_2$ . . . Proven $\therefore T\hat{R}P = \hat{S}$ . . . 3rd ∠ in Δ $\therefore \Delta TPR \parallel\!\mid \Delta RPS$ . . . A ; A ; A $\frac{TP}{RP} = \frac{PR}{PS}$ $PR^2 = PS.TP$ (1) $\frac{QR}{SP} = \frac{RT}{PR}$ proven $\therefore \left(\frac{QR}{SP}\right)^2 = \left(\frac{RT}{PR}\right)^2$ $QR^2.PR^2 = SP^2.RT^2$ $\frac{RT^2}{QR^2} = \frac{PR^2}{SP^2}$ $\frac{RT^2}{QR^2} = \frac{TP.PS}{PS^2}$ $\frac{RT^2}{QR^2} = \frac{TP}{PS}$	✓S + S ✓ S + R ✓ S + R ✓ Ratio ✓ PR^2 ✓ S ✓ S	(7)
			[12]
			GRAND TOTAL: [150]